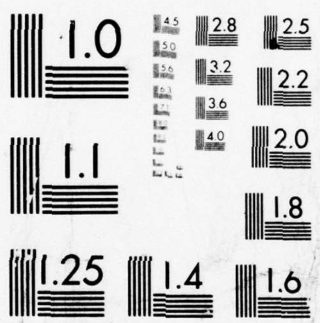
AD-A049 515 MARTIN MARIETTA LABS BALTIMORE MD F/G 20/4
INTERNAL GRAVITY WAVE GENERATION BY VORTEX WAKES.(U)
DEC 77 S C TRAUGOTT F44620-75-C-0009
UNCLASSIFIED MML-TR-77-71C AFOSR-TR-77-1309 NL

1 OF 1
AD
A049515

END
DATE
FILMED
3 — 78
DDC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0

ADA049515

77-1

MARTIN MARIETTA

Martin Marietta Laboratories

MML TR 77-71c

INTERNAL GRAVITY WAVE GENERATION BY VORTEX WAKES

FINAL REPORT

by
S. C. Traugott

DDC
FEB 2 1978

December 1977

Prepared for
Department of the Air Force
Air Force Office of Scientific Research (AFSC)
Bolling Air Force Base, DC 20332

Under Contract F44620-75-C-0009

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 77-1309	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) INTERNAL GRAVITY WAVE GENERATION BY VORTEX WAKES		5. TYPE OF REPORT & PERIOD COVERED FINAL 1 Aug 74 - 30 Sep 77
		6. PERFORMING ORG. REPORT NUMBER MML TR 77-71c
7. AUTHOR(s) S C TRAUGOTT		8. CONTRACT OR GRANT NUMBER(s) F44620-75-C-0009
9. PERFORMING ORGANIZATION NAME AND ADDRESS MARTIN MARIETTA CORP/MARTIN MARIETTA LAB 1450 SOUTH ROLLING ROAD BALTIMORE, MARYLAND 21227		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 2307A1 61102F
11. CONTROLLING OFFICE NAME AND ADDRESS AIR FORCE OFFICE OF SCIENTIFIC RESEARCH/NA BLDG 410 BOLLING AIR FORCE BASE, D C 20332		12. REPORT DATE Dec 77
		13. NUMBER OF PAGES 54
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

VORTEX WAKE
VORTEX VELOCITY
NUMERICAL VISCOSITY
VORTEX INTERACTION
VISCOUS VORTEX DECAY

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This Final Report summarizes progress and accomplishments under AFOSR Contract F44620-75-C-0009. The investigation focused on various details relating to the numerical prediction of vortex wakes. Contributions are discribed to the reduction of false transport effects and the prediction of the descent velocity of an unsteady, viscous, strongly interacting vortex distribution.

14 MML-TR-77-71c

6

INTERNAL GRAVITY WAVE GENERATION BY VORTEX WAKES

9 Final ~~Scientific~~ Report, 1 Aug 74-30 Sep 77

Contract F44620-75-C-0009 15

11 Dec~~ember~~ 1977

12 56p.

Principal Investigator: S.C. Traugott

10 S. C. Traugott

18 AFOSR 19 TR-77-1309 16 2307 17 A1

MARTIN MARIETTA CORPORATION
Martin Marietta Laboratories
1450 South Rolling Road
Baltimore, Maryland 21227

Prepared for
DEPARTMENT OF THE AIR FORCE
Air Force Office of Scientific Research (AFSC)
Bolling Air Force Base, DC 20332

407 998

CONTENTS

Summary of Activity During Contracting Period

This final scientific report summarizes progress and accomplishments under contract F44620-75-C-0009. The period of support under this contract was from August 1, 1974 to September 30, 1977. During this time the investigation focused on various details relating to the numerical prediction of buoyant vortex wakes, and also on the prediction of radiative decay rates of atmospheric waves. A calculation of the complete vortex wake including buoyancy was not completed by the termination date. However, fundamental contributions were made on issues relating to numerical false viscosity effects at large Reynolds number, and to the prediction of the descent velocity of a closely interacting vortex pair. Papers were presented on these topics. These contributions are described in the present report. A manuscript for publication on the latter study is now in preparation. The work on atmospheric radiative decay rate was the subject of an oral paper and has also been published.

Publications and Presentations

1. S. C. Traugott, "Viscous Decay of a Two-Dimensional Interacting Vortex Pair", presented at 1975 Annual Meeting of the Division of Fluid Dynamics, American Physical Society, College Park, Md., November 1975. See Bull. APS, II, 20, 11, November 1975, p. 1428.

2. S. C. Traugott, "Infrared Cooling Parameterization for Atmospheric Perturbations of Arbitrary Size", Proceed. Symp. on Radiation in the Atmosphere, Garmisch-Partenkirchen, August 1976, 498-500, Science Press, Princeton, N. J., 1977.

3. S. C. Traugott, "Infrared Cooling Rates for Two-Dimensional Thermal Perturbations in a Nonuniform Atmosphere", J. Atm. Sci. 34, 6, 864-872, 1977.

4. S. C. Traugott, "Some Examples of Drift Velocity with Two-Dimensional Distributed Vorticity", presented at 1977 Annual Meeting of the Division of Fluid Dynamics, American Physical Soceity, Bethlehem, Pa. See Bull. APS II, 22, 10, November 1977, p. 1278.

5. S. C. Traugott, "Drift Velocity for Two-Dimensional Distributed Vorticity," in preparation.

I. General Vortex Wake Considerations

The original objective of the present investigation concerned the mechanism by which large supersonic aircraft flying in the stably stratified stratosphere would generate atmospheric internal gravity waves. There is no question that such a mechanism exists in principle from two sources: first, the vertical motion of the aircraft vortex wake; second, the ultimate collapse of a well-mixed wake region in stratified surroundings. Both mechanisms are known to exist. The first has been discussed from two essentially different points of view by Saffman[1] and Lissaman et al[2], with contradictory conclusions. The second has received some attention due to its relevance to detection of submerged objects in a stratified ocean based on the emitted waves from wake collapse[3]. With respect to aircraft, prior work did not permit a ready assessment of the efficiency of generation or the magnitude, persistence, and extent of the wave field left behind in the atmosphere. All that can be concluded with certainty from prior studies is that waves will certainly be generated. If their amplitude and extent is or might in the future become significant, then such waves would be of concern both in atmospheric diagnostics dealing with presumably naturally occurring waves and possibly also with respect to aircraft operation.

The intent of this study was to avoid the need for the arbitrary and partly contradictory and inconsistent assumptions which characterize previous theoretical work on stratified vortex dynamics, and base the investigation on numerical, finite difference integrations of the governing partial differential equations. Such an approach avoids the need to make assumptions regarding vortex wake shape, vorticity entrainment or detrainment, and vortex generation due to buoyancy, but rather determines these phenomena as part of the solution.

Finite difference numerical integrations have been successfully applied in calculating the flow field of rising atmospheric thermals, which are analogous to this study in involving coupled effects of vorticity and buoyancy. Similar techniques have also been used to calculate the generation of gravity waves from the collapse of a uniformly mixed region in a stratified atmosphere[4,5,6]. Therefore there is every reason for expecting that modifications and adaptations of known techniques will lead to numerical predictions from which one can assess the magnitude and propagation characteristics of vortex wake generated gravity waves.

At the present termination time of this contract, such predictions have not been achieved. Among the various features of the calculation requiring special treatment, two were particularly troublesome and needed considerably more effort than had originally been anticipated. These were deterioration of numerical accuracy for a reasonable, fixed grid size with increasing Reynolds number, and means of keeping the computing mesh fixed on the vortex wake as it moves vertically at a variable and unknown velocity. Procedures were developed to deal with both problems. They appear to be very promising. However, the opportunity to further develop and apply them to the problem of wave generation by vortex wakes now no longer exists. Both the problems of sufficient accuracy at high grid Reynolds numbers and maintenance of a vorticity-fixed coordinate system are quite general and not only of interest for the particular problem considered here. Because of the interest and importance of these matters, the approach developed under the present contract for dealing with them is described in this report in some detail.

A number of specific vortex dynamics and decay predictions were obtained, both before and after incorporation of the scheme to improve accuracy at large grid Reynolds number. These are all for the isothermal, non-buoyant case. These results are also presented in this report. While the question of the importance of a vortex wake as a generator of gravity waves remains unanswered, it still appears that one way to answer it is to carry to completion the approach taken in this study.

II. Initial Conditions

During an earlier period of AFOSR sponsored research, a method for the numerical finite difference integration of the coupled Navier-Stokes (in conservation of vorticity form) and energy equations was programmed and applied to a problem involving two-dimensional, natural convection[7]. This program was used as a base from which the calculation method for the present problem evolved.

Since the intent was to calculate the dynamics of the late, far downstream stage of wake development and not the early formation stage, an initial flow and temperature field representing a buoyant vortex wake is needed to start the calculation. This should be representative of a vorticity field which is distributed over the wake cross section and not unrealistically concentrated into small vortex "cores".

The choice for a starting velocity field was the inviscid, steady state vorticity distribution given in Lamb[8] and also discussed by Batchelor[9]. This two-dimensional flow is the analog of the well-known Hill's spherical vortex. It can be thought of as representing a steadily downward moving, inviscid vortex wake in coordinates moving vertically with the wake, in a

plane located at a large fixed distance behind an aircraft.

The flow is given by

$$\psi = -\frac{2VJ_1(kr)\sin\theta}{k\,J_0(ka)}\,, \qquad r \leq a$$

$$\psi = -V\left(r - \frac{a^2}{r}\right)\sin\theta\,, \qquad r \geq a$$

Here ψ is a stream function, V is a constant reference velocity, r and θ are cylindrical coordinates with $\theta = 0$ pointing vertically downwards. J_0 and J_1 are Bessel functions, of order zero and one, and k is a constant given by the zero of J_1 which defines the radius of a circle which confines the region of distributed vorticity. Thus

$$ka = 3.83171$$

$$J_0(ka) = -0.402759$$

The stream function above gives an inner flow whose vorticity Ω is proportional to ψ itself, through

$$\Omega = k^2\psi$$

Therefore the vorticity vanishes on the bounding circle r = a, while beyond it the flow is irrotational. The inner flow has interior stagnation points separated by the distance 0.961023a where vorticity and stream-function have a maximum. Thus it represents the vorticity distribution of an interacting vortex pair, sinking downwards with velocity V, in a coordinate system fixed upon this vortex pair. This velocity is given by $V = \Gamma/6.8339a$, where Γ is the circulation about half the flow, i.e. $\Gamma = \int_{-\infty}^{\infty}\int_{0}^{\infty} \Omega dxdy$. There are no infinite velocities anywhere. Inner and outer flow match in velocity, vorticity, and shear. The inner flow is an exact solution of the inviscid, unsteady equations. The action of viscosity

causes the interior vorticity to decay and to spread beyond the initial confining boundary given by $r = a$.

These features are illustrated in Fig. 1. The upper part of the figure shows streamlines on the left, contours of constant vorticity on the right. These results come from our computer program; they represent the theoretical solution just described but obtained from a finite difference integration of several time steps after an initial input of the theoretical solution. In such a starting procedure, viscosity is taken as zero in the calculation. The lower part of the figure shows the horizontal distribution of vertical velocity v on the left, and the vorticity Ω on the right. These quantities have been normalized by

$$v = \frac{v'H}{\nu} \qquad \Omega = \frac{\Omega' H^2}{\nu}$$

Here v' and Ω' are dimensional, and ν is kinematic viscosity. H is the size of the square computing region comprising half the flow field, and this is four times the initial radius of the vortex bubble. The solid lines reoresent the initial, inviscid distributions.

The dotted curves indicate the result of a viscous calculation carried out to a later time. The expected re-distribution of vorticity and attenuation of vertical velocity is evident. A dimensionless time is defined by $t' = \frac{\nu t}{H^2}$.

For the non-isothermal case, no computations were made but an analogous temperature field to start the calculation has been formulated.

The initial velocity field satisfies the appropriate steady inviscid equation of motion

$$\frac{\partial\psi}{\partial y}\frac{\partial\nabla^2\psi}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\nabla^2\psi}{\partial y} = 0$$

through

$$\nabla^2\psi = -k^2\psi \ .$$

The steady, non-conducting version of the energy equation admits solutions of the same kind through the more general form

$$\phi = \phi(\psi)$$

where $\phi = T_s + \Delta T + \gamma y$ is the sum of ambient temperature T_s, an unknown temperature perturbation ΔT to be solved for, and compressibility in the ambient atmosphere represented by the adiabatic lapse rate γ. A particular case would be $\phi = -k^2\psi$, in which lines of equal potential temperature would be identical to lines of constant vorticity. This is not an appropriate starting solution, since it corresponds to a vortex pair, one of which is hotter and the other colder than ambient. A more appropriate solution appears to be

$$\phi = \phi_0\psi^2 \, .$$

This function represents hot spots centered on the vortices vanishing smoothly at the initial wake boundary $r = a$ where $\psi = 0$. The temperature excess scales with ϕ_0. This function would serve as an initial input to the energy equation, with ϕ_0 as a disposable parameter.

III. Boundary Conditions

The flow illustrated in Fig. 1 has the property that far from the origin the velocity asymptotically becomes a spatially constant vertical updraft with magnitude V. This boundary condition cannot be enforced on the finite boundaries of the computing region which surrounds the vortex pair, since an upper bound exists on the size of this control surface due to computing cost limitations. Typically, the computing region cannot be larger than an order of magnitude greater than a. The problem is quite common and occurs whenever boundary conditions at infinity need to be

imposed with finite difference calculations and finite budgets. The following scheme was developed to deal with this problem.

The interior vorticity distribution, whatever it may be, determines the velocity at arbitrary points x_b, y_b through the appropriate Green's function:

$$u_b(t) = -\frac{2}{\pi}\int_0^H\int_0^X \Omega(x',y',t)\,\frac{(y_b-y')x_b x'dx'dy'}{[(x_b-x')^2+(y_b-y')^2]\,[(x_b+x')^2+(y_b-y')^2]}$$

$$v_b(t) = V(t) + \frac{1}{\pi}\int_0^H\int_0^X \Omega(x',y',t)\,\frac{[x_b^2-x'^2-(y_b-y')^2]x'dx'dy'}{[(x_b-x')^2+(y_b-y')^2][x_b+x')^2+(y_b-y')2]}$$

Here x' and y' are coordinates of arbitrary interior points where the vorticity is Ω. Integration over all such points contained in a rectangular control surface specifies the velocity at some point on this surface (x_b,y_b), where vertical and horizontal boundaries are given by H and X. The above expressions have been incorporated into our numerical program, and they evaluate boundary velocities at any time based on the vorticity distribution which has been determined at that time. These boundary velocities are then related to boundary stream-functions which are used to find interior velocities used to go on in time.

This Green's function formulation for velocity boundary conditions has worked very well. Accuracy as measured by comparing the numerically calculated inviscid flow including boundary values after a few time steps with the exact analytical results for this flow is satisfactory. This formulation, being kinematic, is equally valid for isothermal and buoyant calculations.

In a stratified ambient environment, the vortex wake will appear to move through a time-dependent temperature field if observed in a coordinate system fixed on the wake. If T_S is the temperature at any height far from the wake, then at a point there fixed relative to the wake one will see a temporal temperature change given by

$$\frac{\partial T_S}{\partial t} = V\left(\frac{dTs}{dy} + \gamma\right)$$

Here γ is the adiabatic lapse rate. This will be the appropriate temperature boundary condition. This time-varying temperature can be applied, without serious error, on the boundary of the computing region, in contrast to the case for velocities. This boundary condition introduces yet another role for $V(t)$. For positive stratification $\left(\frac{dTs}{dz} + \gamma > 0\right)$, a downward moving vortex wake ($V<0$) will sense an ever cooler environment and become ever more buoyant. Thus V attains great dynamical significance for the non-isothermal case.

IV. Conventional Finite Difference Method Results

The finite difference scheme initially utilized is standard and only the treatment of the non-linear convection terms requires comment. A particular three-point, non-central operator proposed by Torrance[10,11] was used. It employs forward or backward differences depending on the direction of the local velocity. The operator has the property that it satisfies conservation requirements and imposes no grid size restriction to prevent numerical instability. These strong advantages need to be balanced against the disadvantage that the resulting accuracy is only first order in grid size, in contrast to second order accurate three point central differences.

One way to view this matter is that with non-central differences, results can be obtained which may become inaccurate at too coarse a grid; with central differences, results cannot then be obtained at all due to numerical instabilities. The general impression regarding the resulting finite difference equations, gathered at the time this study began from the literature and personal communications, was that while formally some inaccuracy problems were to be expected at large grid Reynolds number; in practice these were not serious.

Results obtained with this method for the isothermal wake will now be briefly described.

Two runs were made. The values of the Reynolds number were 5 and 250. This Reynolds number is based on the initial downward vortex wake velocity and the initial spacing between the vortex centers as given by the wing span. If the latter is 50m, and the downward velocity is 1 $msec^{-1}$ (corresponding to an aircraft speed of 300 $msec^{-1}$ with typical cruise lift conditions), then the corresponding values for viscosity are 10^5 and $2 \times 10^3 cm^2 sec^{-1}$, respectively. The first represents a large value typical of large scale eddy mixing in the troposphere, the second is more typical of turbulence generated within the wake itself.

The initial flow configuration has already been shown in Fig. 1. At the small Reynolds number, streamlines and vorticity 100 time steps into the calculation are shown in Fig. 2. This time corresponds to a downward motion of the wake of 10% of its original diameter. It has grown substantially larger, but without strong distortion of the bounding streamline. For the large Reynolds number, 240 time steps result in the flow shown in Fig. 3. By this time the wake has moved downwards just under 1.5 initial

diameters. It is now significantly distorted, as shown both by the vorticity pattern and bounding streamline. A composite illustration showing the progressive distortion of the bounding streamline as a function of time, for both Reynolds numbers, is given by Fig. 4, where the dots near the center indicate the outward movement of the vortex "center". This distance is used as a measure of wake size. The wake boundary is superimposed on vorticity contours at the largest time for each run, in Fig. 5. This illustrates the beginning development of a vortex wake when the Reynolds number is large.

As the wake grows and distorts, its downward velocity also changes. For these examples it decreases, so that the wake slows down. For both cases, the runs were of such duration that the wake drift velocity decreased to about 50% of its initial value.

A particular representation of this phenomenon is shown in Fig. 6. The ordinate in both plots is a special combination of downward velocity, wake size, and wake impulse suggested by an analysis due to Maxworthy[12]. The quantity I represents a measure of the impulse needed to generate the motion from rest. Here, it has been evaluated from $I = (\frac{b}{2})^2 V$, with b the instantaneous vortex spacing. The combination of variables making up the ordinate of Figs. 8 and 9 is one which is predicted in reference 12 to be linearly proportional to time. It was found possible to determine an effective origin in our examples such that, very nearly, the predicted linear variation is observed.

Two aspects of Fig. 6 need further discussion. First, the Maxworthy[12] prediction is not only a linear variation with time of the ordinate of Fig. 6, but, in our present non-dimensional variables, also a slope which

is independent of Reynolds number. This is not found to be the case with the initial numerical calculations. It was not clear whether this Reynolds number effect on slope is just a consequence of wake distortion, which would influence wake drag and decay and is not included in Maxworthy's analysis, or numerical errors.

The other matter concerns the obvious starting transients visible in the data. A modification has been made subsequent to the generation of the data shown, in which a new initial step removes the discrepancy between initial analytical data and a slightly inaccurate version of this same data, which is produced by numerical relaxation.

V. Modification to Reduce False Transport Effects

As the investigation proceeded, several indications began to suggest that all was not well with the high Reynolds number calculation. First, for this condition it was found that the downward drift velocity would still decay significantly even with deletion of viscosity. Second, the decay was significantly reduced by changing the computing mesh from 60x60 to 120x120. While the disagreement regarding the Maxworthy prediction of Reynolds number independence was also disquieting, this disagreement was somewhat ambiguous because the latter prediction is approximate and not intended for Reynolds numbers as low as those considered here. Finally, other assessments regarding the Torrance convective operator appeared[13] which gave definite indication that the Reynolds number of 250 calculation was probably seriously contaminated by false transport effects. For the coarse mesh (60x60), our highest grid Reynolds number was approximately 10, which should now, in light of present insight, be considered unacceptably large. It was realized

that if false transport effects were present, these would seriously affect subsequent gravity wave predictions, due to their expected relatively fine spatial structure.

The following scheme was designed in an attempt to eliminate first order grid size errors without going to central differences, thereby hopefully avoiding the need for ever decreasing mesh size with increasing Reynolds number to maintain numerical stability. To my knowledge, this very simple idea has not been explored before.

From a Taylor series expansion of the function whose derivative is to be approximated, one can obtain for the point $x=x_i$, where $u=u_i$:

$$\frac{\partial u\Omega}{\partial x}_{FD} = \frac{\partial u\Omega}{\partial x}_{DIF} - \frac{\Delta x}{2}\left[u\frac{\partial^2\Omega}{\partial x^2} + \frac{\partial u}{\partial x}\frac{\partial\Omega}{\partial x}\right]_i + O(\Delta x^2)$$

$$\frac{u_{i+1}+u_i}{2} > 0, \frac{u_i+u_{i-1}}{2} > 0$$

$$\frac{\partial u\Omega}{\partial x}_{FD} = \frac{\partial u\Omega}{\partial x}_{DIF} + \frac{\Delta x}{2}\left[u\frac{\partial^2\Omega}{\partial x^2} + \frac{\partial u}{\partial x}\frac{\partial\Omega}{\partial x}\right]_i + O(\Delta x^2)$$

$$\frac{u_{i+1}+u_i}{2} < 0, \frac{u_i+u_{i-1}}{2} < 0 .$$

Here u is the horizontal velocity in the x direction, and Ω is the vorticity. The subscript FD denotes finite difference, DIF denotes differential. These particular forms result from the special difference operators used here[10,11]. If opposite signs occur for the mean velocities at adjacent gridpoints, no such simple expression appears to be derivable. For that case, the approximation is suggested that the first order terms should be neglected entirely. Corresponding expressions can be derived for the vertical convective transport terms.

The equations above are not new. The first order difference between finite difference and differential derivatives is usually called false viscosity when u or v is constant, here they are simply referred to as first order errors. Let all such terms be lumped together as $E\Delta x$.

If the remaining terms in the original unsteady differential equation for vorticity are similarly expanded, one finds that the difference equation which is to be solved numerically, which has the form

$$\text{FD equation} = 0 \tag{1}$$

becomes

$$\text{DIF equation} \mp E\Delta x + \frac{\Delta t}{2}\frac{\partial^2 \Omega}{\partial t^2} + O(\Delta x^2) = 0 \tag{2}$$

The interpretation of (2) is that, to $O(\Delta x)$, the differential equation is not satisfied if one has programmed (1). But this is only a question of book-keeping. The term $E\Delta x$ can be attached to (1) rather than (2) if one considers the difference equation to be

$$\text{FD equation} = \mp E\Delta x \tag{3}$$

One would then find

$$\text{DIF equation} + \frac{\Delta t}{2}\frac{\partial^2 \Omega}{\partial t^2} + O(\Delta x^2) = 0 \tag{4}$$

The second order error in time is found to be negligible for the problem dealt with here.

Equation (3) can be interpreted as the result of constructing a finite difference approximation not of the real differential equation but of an artificial one in such a way that the finite difference version of the artificial equation is, to second order accuracy, the proper differential equation.

Since the quantity $E\Delta x$ involves various space derivatives, these are known or can be operated with in exactly the same way as the differential operators already occurring in (1) or the left side of (3). A scheme to improve accuracy therefore is to program and apply (3) rather than (1). This modification to our original program has been made and tested in a limited way, with the following results.

For an inviscid, unmodified run (all viscous terms removed from the governing equation), one finds with the normal 60 x 60 grid a flow decay of 0.394%. With modification it is 0.005%. In this example the correct decay is known; it is no decay at all. The modified program does better by a factor of 79, which is of the order expected in going to Δx^2 accuracy compared to Δx.

Some tests were also made for the viscous, large Reynolds number case described in the previous section. Here comparisons were made between modified and unmodified program, both with a normal grid and a grid half that size. The modification produced a decay which was not significantly affected by halving the grid size. This decay is slower than that with the unmodified program and the original grid size, and differs even more from the lower Reynolds number case than was the case with the unmodified program.

Examination of the flow field predicted at the large Reynolds number with the modified program shows the development of small oscillations in the vorticity contours. This is illustrated in Fig. 7. This configuration exists after 60 time steps. It is not known at this time if this result is real, and the possibility needs to be examined that a

flow instability is developing at this fairly large Reynolds number. The most obvious test is to repeat the calculations with a finer mesh; such calculations have not yet been made. No such oscillations develop with the unmodified program and enhanced false viscosity effects, or at the low Reynolds number.

VI. Theoretical Analysis of Vertical Wake Velocity

The instantaneous overall vertical velocity V plays several roles. It is an a priori unknown variable in time and one of the main characteristics of the vortex wake to be found. It is also needed to keep the computing grid centered on the vortex system. Without such an arrangement the region of interest would soon escape any computing grid of reasonable size and resolution. Finally, it is required for the temperature boundary condition, as described in Section III.

A numerical control system was constructed which monitors the interior stagnation point of the flow field for vertical drift of the calculated flow with respect to the coordinate system supposedly moving with the vortex system. A correction was then calculated from this data and applied to V for the succeeding time step to prevent this drift. The correction was designed to so alter V, at each time step, that net drift is prevented, to a tolerance, for all time. The resulting V(t) is then the instantaneous vertical downward velocity of the vortex wake which is sought.

Some problems were encountered in predicting the evolution of V(t). An initial, crude version of a control system for drift prevention was too insensitive and produced no control; a second version was much too sensi-

tive and produced unstable oscillations. Several succeeding formulations all failed to damp these oscillations.

Finally, however, a scheme was found which seemed to be both stable and effective in preventing drift. This consisted of three separate contributions to a corrective velocity which are proportional to stagnation point displacement, displacement rate, and the time derivative of the latter. Judicious choice for the coefficients of these corrections was able to produce a non-oscillatory V(t) with the unmodified, original computer program, and this scheme was used to produce the results described in Section IV. However, the situation still was not very satisfactory. A rational and reliable means for determining the coefficients was never found. Further, there were cases in which oscillations would begin again after a considerable computing time with smooth decay, and then new coefficients would have to be found, by trial and error.

When the computer program included the modifications described in Section V to remove false transport effects, the problem of oscillations in the predicted V(t) returned. At a Reynolds number of 250, coefficients in the correction to V were never found to truly stabilize the motion of the coordinate system. This is indicated in Fig. 8.

In the isothermal case this situation is unfortunate but not one to entirely prevent proper interpretation of calculated results. In effect, one views the flow field as one might a television screen picture with oscillations in the vertical hold control. At any instance, the spatial picture is correct, and the oscillation is an annoyance. Since the wake velocity V should almost certainly decay in a non-oscillatory manner, one can even still extract V from some averaging process. However, with a buoyant wake in a stratified atmosphere one of the primary questions is

whether the entire wake will oscillate, and how much. Then the situation just described is entirely unacceptable. Therefore a different method for determining the quantity V(t) had to be found. This section briefly describes the results of work done on this problem.

The approach taken was to search for a theoretical basis for determining the overall velocity of a region of distributed vorticity, allowing for viscous and non-steady effects. No derivation of a theoretical expression for velocity was found in the literature, nor was our attempt at this successful.

Even for the incompressible, isothermal case, only Saffman[14] appears to have seriously addressed the problem. Although Saffman's study [14] is applied to a viscous vortex ring of small cross-section relative to diameter, he also gives a general definition of vortex system velocity. This velocity is defined rather than derived, but he gives supporting arguments for the definition. Therefore Saffman's definition, as well as some others based on intuition or found in the literature, was calibrated on two cases for suitability of incorporating such theoretical expressions into the computer program. This calibration leads to the conclusion that Saffman's definition appears to be correct and suitable, as will now be described.

Consider a general vorticity distribution $\Omega(x,y)$ which is antisymmetric about the vertical y axis. For the special case of our starting solution, there is also symmetry about the horizontal x axis, but with time this disappears under the action of viscosity. In this analysis, the variables have not been normalized.

The quantity $\iint_{-\infty}^{\infty} \Omega dxdy$ is preserved for all time in any two-dimensional boundary-free viscous flow; unfortunately here this quantity is zero.

It can be shown that the non-vanishing integral corresponding to the vertical impulse is also preserved. This is given by

$$I_y = -\frac{1}{2}\iint_{-\infty}^{\infty} x\Omega dxdy \tag{5}$$

A similar invariant integral corresponds to the horizontal impulse I_x; for the present case this is zero.

Saffman[14] argues that the vertical velocity of the entire vorticity distribution is given by

$$V = \frac{1}{2}\frac{d}{dt}\iint_{-\infty}^{\infty}\frac{\left(y^2 I_x - xyI_y\right)}{I_x^{\,2}+I_y^{\,2}}\,\Omega dxdy \tag{6}$$

For the present case, this reduces to

$$V = -\frac{1}{2I_y}\iint_{-\infty}^{\infty} xy\,\frac{\partial\Omega}{\partial t}dxdy \tag{7}$$

The time derivative represents the variation with time of the vorticity distribution in a coordinate system fixed in space (not on the vortex system), at a fixed position.

The dynamic equation governing the evolution of vorticity can now be used to evaluate $\frac{\partial\Omega}{\partial t}$.

For the isothermal, incompressible case (7) can be written as

$$V = \frac{1}{2I_y}\iint_{-\infty}^{\infty}\left[\frac{\partial}{\partial x}(u\Omega) + \frac{\partial}{\partial y}(v\Omega) - \nu\nabla^2\Omega\right]xydxdy \tag{8}$$

It can be shown, after some manipulations, that for the present case (8) can eventually be written as

$$V = \frac{\iint_{-\infty}^{\infty} (xv + yu)\Omega dxdy}{\iint_{-\infty}^{\infty} x\Omega dxdy} \tag{9}$$

Various terms arising from integration by parts, and evaluated far away from the region near Ω, vanish because of the anti-symmetrical nature of the vorticity distribution.

Once the definition of vortex system velocity given by Saffman[14] has been manipulated into the form (9), direct comparison with other definitions becomes possible.

Not only does the quantity $\iint_{-\infty}^{\infty} \Omega dxdy$ vanish, but so does $\iint_{-\infty}^{\infty} v\Omega dxdy$. This suggests the definition

$$V = \frac{\int_{-\infty}^{\infty}\int_{0}^{\infty} v\Omega \; dxdy}{\int_{-\infty}^{\infty}\int_{0}^{\infty} \Omega \; dxdy} \tag{10}$$

Lo and Ting[15] claim the equivalent of (10) to be the velocity of the vortex system. A similar form is used by Bilanin et al[16].

The fact that the denominator of (10) will generally be time dependent may be a worry to some, and an obvious remedy is to define

$$V = \frac{\iint_{-\infty}^{\infty} xv\Omega dxdy}{\iint_{-\infty}^{\infty} x\Omega dxdy} \tag{11}$$

This intuitive definition is recognized as the first term of (9).

The velocities from (9), (10), and (11) will be denoted by V_9, V_{10}, and V_{11}. It is straightforward to evaluate them for the steady, inviscid distributed vorticity which has been the starting flow of our numerical computations. It should be remembered that all integrands are referred to coordinates fixed in space.

If the known steady velocity in this case is V_0 (see Section II), then one finds

$$V_9 = V_{10} = 2V_{11} = V_0.$$

In other words, both terms of (9) are necessary, and both Saffman's[14] and the more conventional definition (10) are correct for this inviscid steady flow.

A second comparison of the various expressions for V is with the viscous, unsteady flow configuration of two counter-rotating Lamb vortices. The evolution of this configuration was also calculated by Bilanin et al[16], using a numerical finite difference model including turbulence. Here the approach is laminar, and analytical predictions can be obtained.

Let the vorticity be

$$\Omega = \Omega_R + \Omega_L$$

with

$$\Omega_R = \frac{\Gamma}{4\pi\nu t} \exp - \left[\frac{(x-\frac{b}{2})^2 + y^2}{4\nu t} \right]$$

$$\Omega_L = \frac{-\Gamma}{4\pi\nu t} \exp - \left[\frac{(x+\frac{b}{2})^2 + y^2}{4\nu t} \right]$$

Thus two vortices of opposite sign are located at $x = \pm\frac{b}{2}$, $y=0$, separated in x by the distance b. The initial circulation about each is $\pm\Gamma$.

Whatever the separation, early in time the vortices do not interact since their viscous cores ($\propto\sqrt{\nu t}$) are much smaller than b. At large times they will strongly interact, each attenuating the vorticity of the other. In evaluating the various integrals, at each time t the spatially fixed coordinate system is taken with the x axis coincident with the instantaneous vertical vortex center position.

As $t \to o$, $b^2/4\nu t \to \infty$, the vortex cores become δ functions and $V_9 = V_{10} = V_{11} = \Gamma/2\pi b$, the latter quantity being the correct, known steady velocity for that case.

As $t \to \infty$, $b^2/4\nu t \to o$, one finds

$$V_9 = \frac{1}{8}\,\frac{\Gamma b}{4\pi\nu t} \tag{12}$$

$$V_{10} = .1381\,\frac{\Gamma b}{4\pi\nu t} \tag{13}$$

$$V_{11} = \frac{1}{2}\,V_9$$

In both the present examples the two contributions to the numerator of (9) were found to be equal, explaining why $V_{11} = \frac{1}{2}V_9$. This is probably due to the special nature of the test flows: both maintain vertical symmetry about the x axis, without the trailing tail of detrained vorticity which developed in our numerical computations and also known to exist in traveling viscous vortex configurations generally.

The finding that vortex system velocity is proportional to the quantity $\Gamma b/\nu t$ is believed to be new. It is characteristic of the very last stage of vortex interaction, and differs (but only slightly) from that predicted by Maxworthy[12] for an earlier stage of wake decay. To facilitate

comparison, the present prediction in the dimensionless variables of Fig. 6 is

$$V \, I^{-1} \alpha \, t.$$

The constant of proportionality given by (12) differs from that in (13). The correct theoretical value is not known. All one can say is that there are supporting arguments in favor of V_9 while none seem to exist for V_{10}.

Equation (9) is attr.ctive for other reasons besides a veneer of theoretical rigor. The integrands in both numerator and denominator tend to zero as the y axis is approached (Ω, x, and u vanish there) more rapidly than those in (10). As mentioned before, there will in general be a vertical trail of detrained vorticity located downstream of the vorticity distribution and centered about the y axis. This must eventually cross one of the horizontal boundaries of a vortex-fixed computing region and be lost to the computation, thereby introducing errors. Use of (9) will minimize these.

The conclusion of this part of the investigation is that for the isothermal case equation (9), expressed in vortex fixed coordinates, should be incorporated into the computer program and used as a basis for adjustments of the coordinate system velocity. For the general variable density case, Saffman's definition[14] of velocity might still serve as a base for deriving a generalization to (10). This expectation is based on the fact that I_y still retains its incompressible form in the Boussinesq approximation, as shown by Saffman[1].

VII. Recommendations for Future Work

1. Equation (9) should be incorporated into the computer program as a means of adjusting the velocity of the vortex-fixed coordinate system.

2. The incompressible form of (9) should be generalized to include variable density, utilizing the Boussinesq approximation.

3. The scheme for eliminating first order (in grid size) computing errors should be further tested. This aspect of the investigation is potentially the most valuable and useful, as it extends beyond the confines of the particular problem studied here. Tests to separate real from computational instability can be made with the present problem through a sequence of runs at progressively smaller grid size, and also by constructing numerical solutions for other known, stable flows.

4. The effectiveness by which a strong vortex wake in a stable atmosphere generates internal gravity waves should also be examined from a physical, less "brute force" point of view, to complement any future studies such as these. The present study has not led, during the contracting period, to an evaluation of this mechanism. At this time it is neither possible to dismiss it, nor to advocate it as a strong potential influence on the upper atmosphere or on aircraft flying there.

ACKNOWLEDGEMENTS

The programming and numerical work carried out under this contract would have been impossible without the outstanding abilities of Ms. S. Yamamura during the early phase of the work and, subsequently, those of Mr. Carl Hutton. I wish to acknowledge the courage, support, and understanding of the contract monitor, Milton Rogers.

References

1. Saffmann, P. G., "The Motion of a Vortex Pair in a Stratified Atmosphere", Studies in Applied Mathematics 51, 2, 107-119, 1972.

2. Lissaman, P.B.S., Crow, S.C., McCready, P.B. Jr., Tombach, I.H., and Bate, E.R. Jr.," Aircraft Vortex Wake Descent and Decay under Real Atmospheric Effects", AeroVironment Report No. FAA-RD-73-120, AD 771-311/8, 1973.

3. Schooley, A. H., and Hughes, B.A., "An Experimental and Theoretical Study of Internal Waves Generated by the Collapse of a Two-Dimensional Mixed Region in a Density Gradient", J. Fluid Mech. 51, 1, 159-175, 1972.

4. Wessel, W.R., "Numerical Study of the Collapse of a Perturbation in an Infinite Density Stratified Fluid", Phys. Fluids, 12, 12II, 171-176, 1969.

5. Young, J. A., and Hirt, C. W., "Numerical Calculations of Internal Wave Motions", J. Fluid Mech., 56, 2, 256-276, 1972.

6. Orlanski, I., and Ross, B., "Numerical Simulation of the Generation and Breaking of Internal Gravity Waves", J.G.R., 78, 36, 8808-8826, 1973.

7. Traugott, S. C., and Yamamura, S.H., "Buoyant Convection in Radiating Horizontal Fluid Layers", AIAA J., 13, 1, 88-94, 1975.

8. Lamb, H., "Hydrodynamics", 6th Edition, Dover Publication, p. 245, 1932.

9. Batchelor, G. K., "An Introduction to Fluid Dynamics", Cambridge University Press, p. 535, 1967.

10. Torrance, K.E., "Comparison of Finite Difference Computations of Natural Convection", J. Res. N.B.S., 72B, 4, 281-300, 1968.

11. Torrance, K. E., and Rockett, J. A., "Numerical Study of Natural Convection", J. Fluid Mech., 36, 1, 33-54, 1969.

12. Maxworthy, T., "The Structure and Stability of Vortex Rings", J. Fluid Mech., 51, 1, 15-32, 1972.

13. DeVahl Davis, G., and Mallinson, C. D., "An Evaluation of Upwind and Central Difference Approximations by a Study of Recirculating Flow", Computers and Fluids, 4, 29-43, 1976.

14. Saffman, P. G., "The Velocity of Viscous Vortex Rings", Studies in Appl. Math., XLIX, 4, 371-380, 1970.

15. Lo, K. C., and Ting, L., "Studies of the Merging of Vortices", Phys. Fluids, 19, 6, 912-913, 1976.

16. Bilanin, A. J., Teske, M. E., and Williamson, C. G., "Vortex Interactions and Decay in Aircraft Wakes", AIAA J., 15, 2, 250-260, 1977.

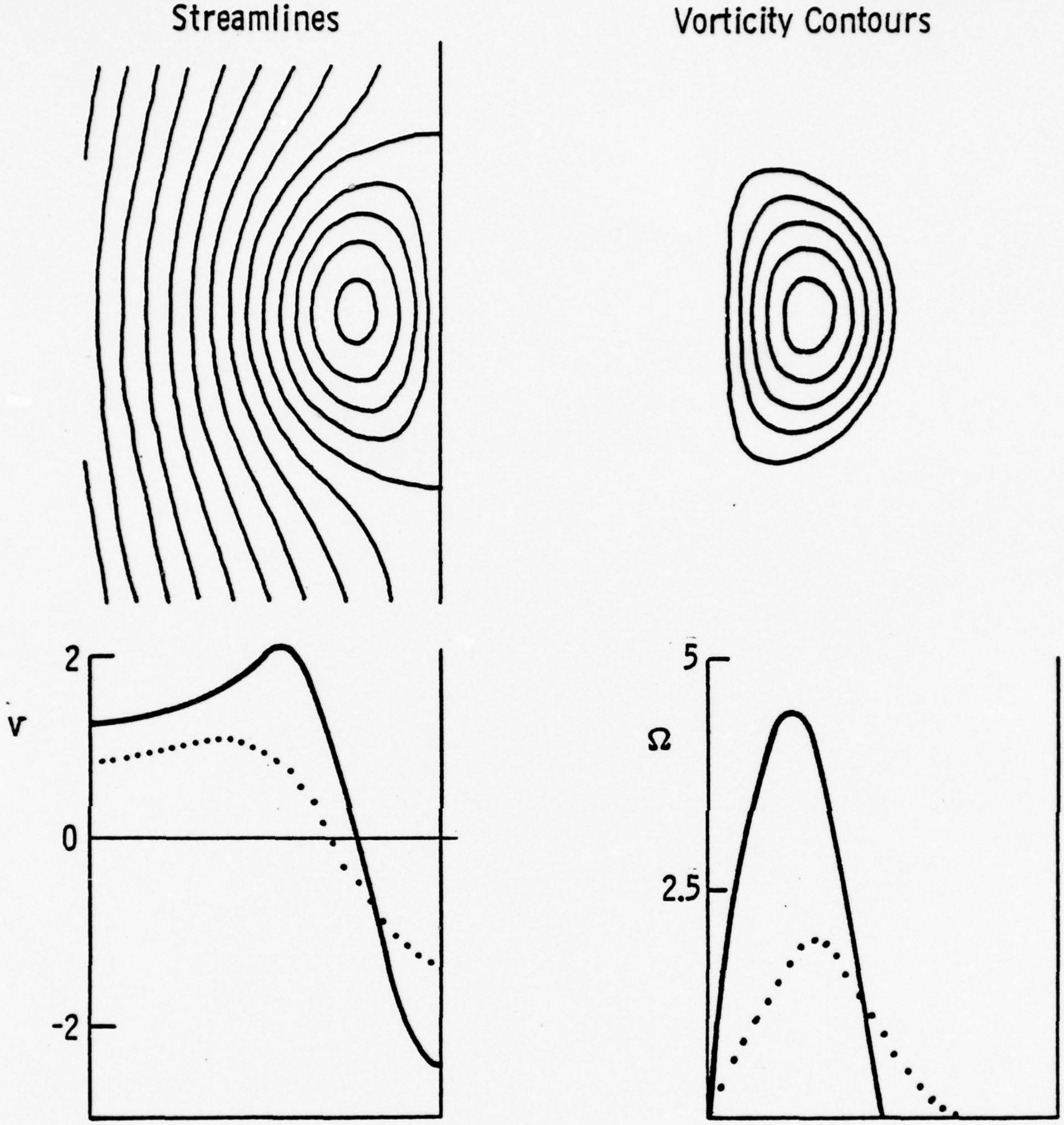


Fig. 1 WAKE FLOW CONFIGURATION

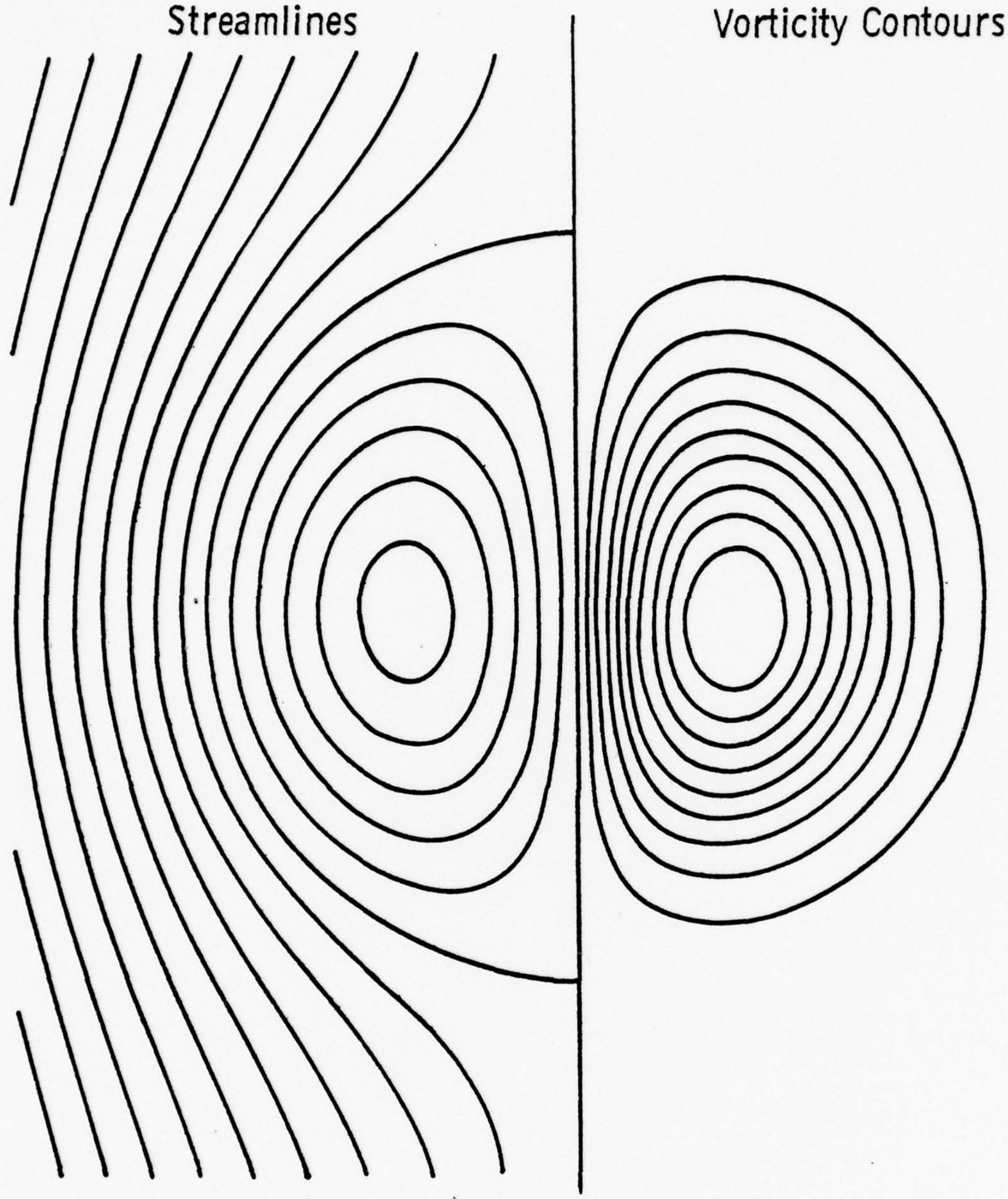


Fig. 2 Reynolds Number = 5, 100 time steps.

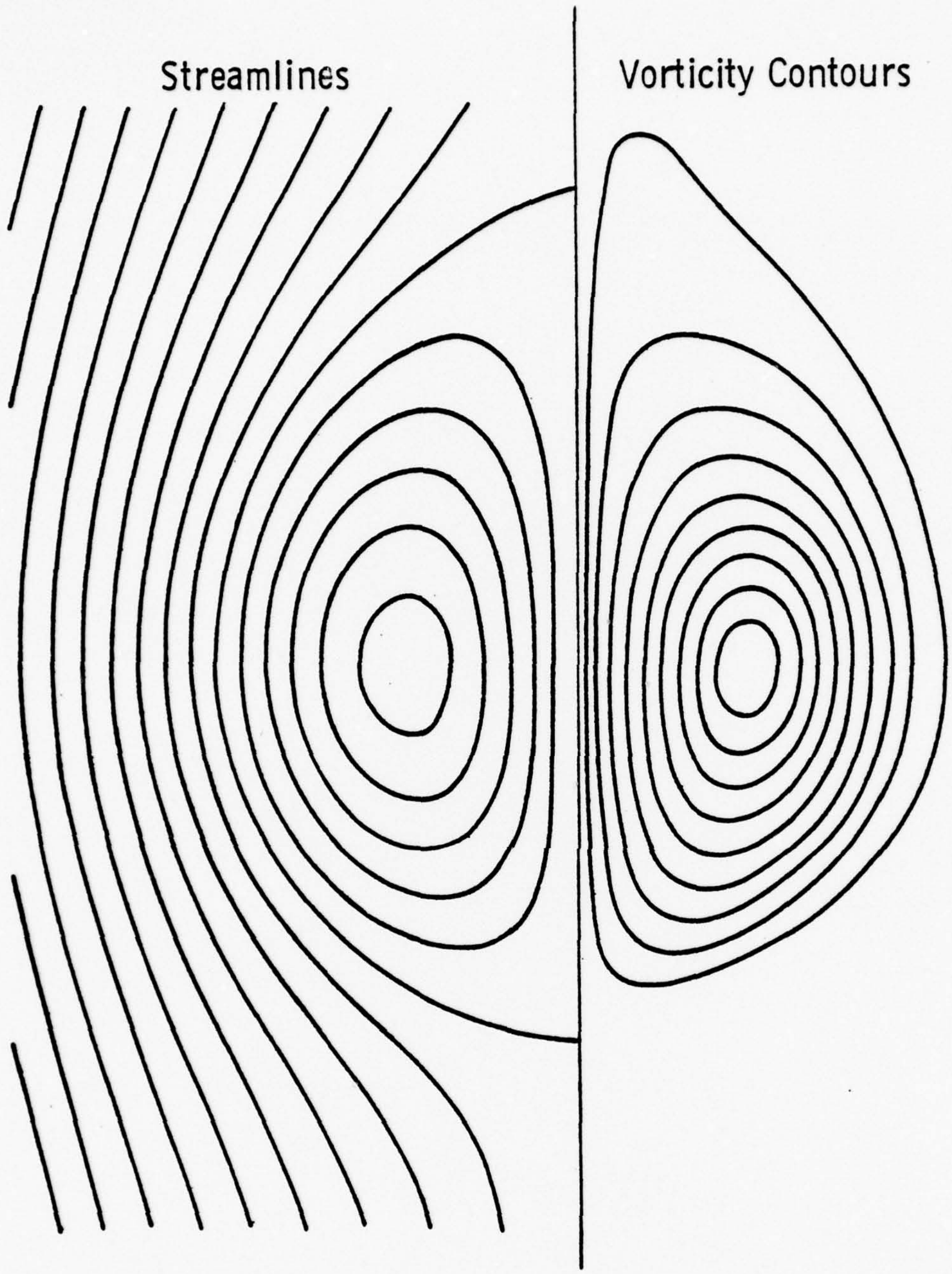
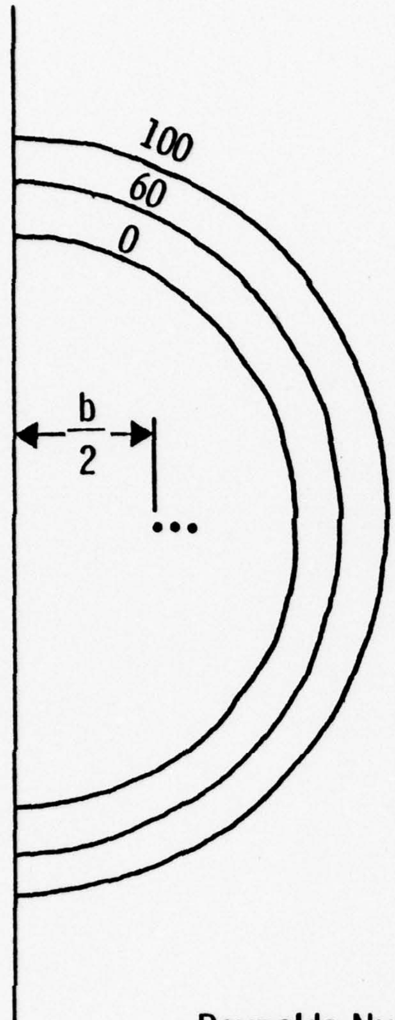
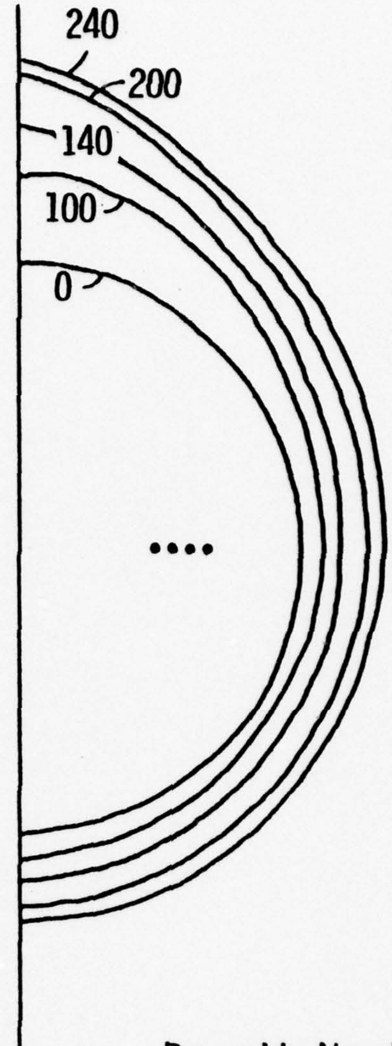


Fig. 3 Reynolds Number = 250, 240 time steps.



Reynolds Number = 5



Reynolds Number = 250

Fig. 4 Growth of $\psi = 0$ Streamline

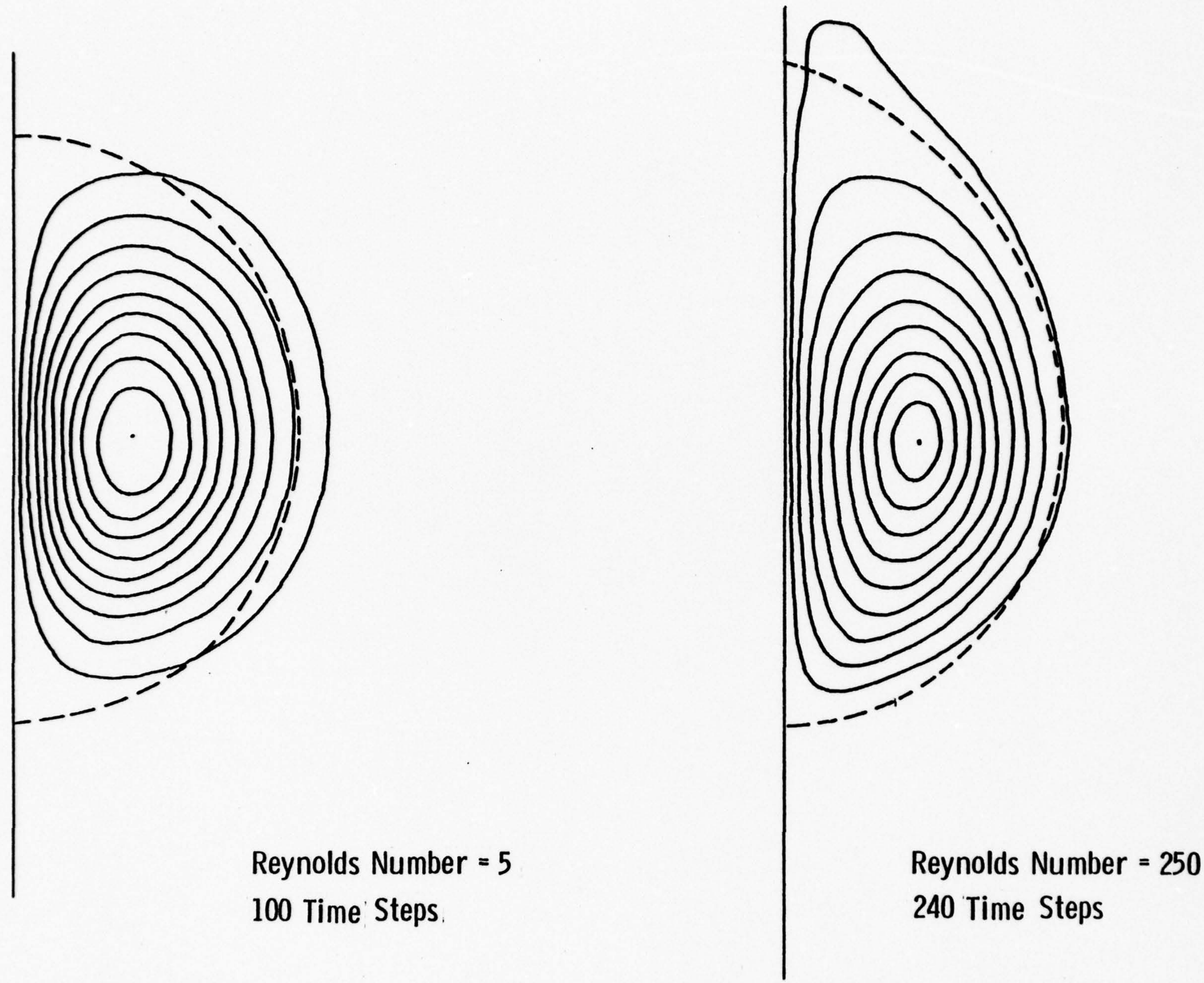


Fig. 5 Velocity Entrainment and Detrainment Relative to $\psi = 0$ Streamline.

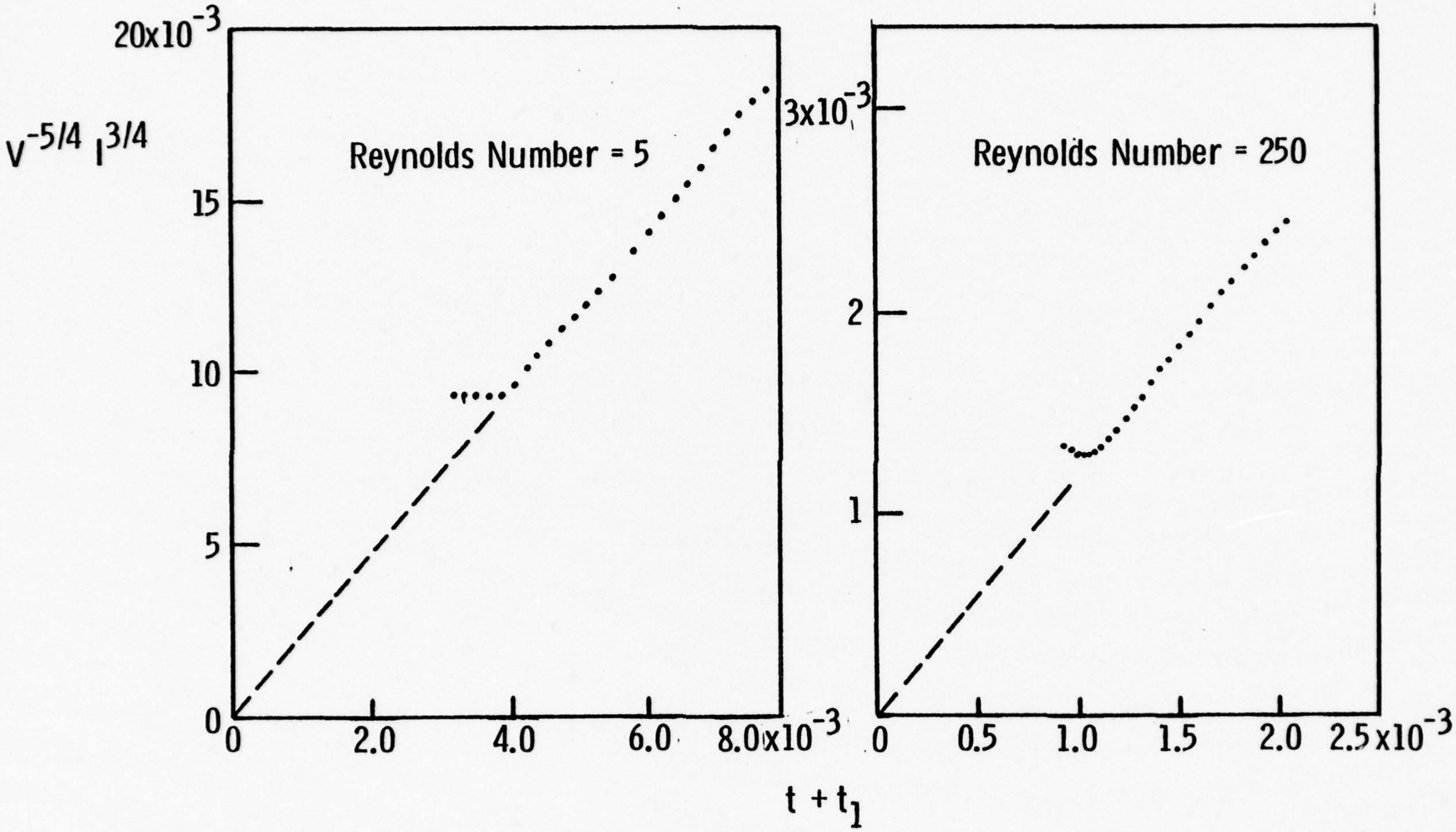


Fig. 6 Vertical Wake Velocity Decay.

Fig. 7 Vorticity Contours, Reynolds Number = 250
60 time steps, Modified Program.

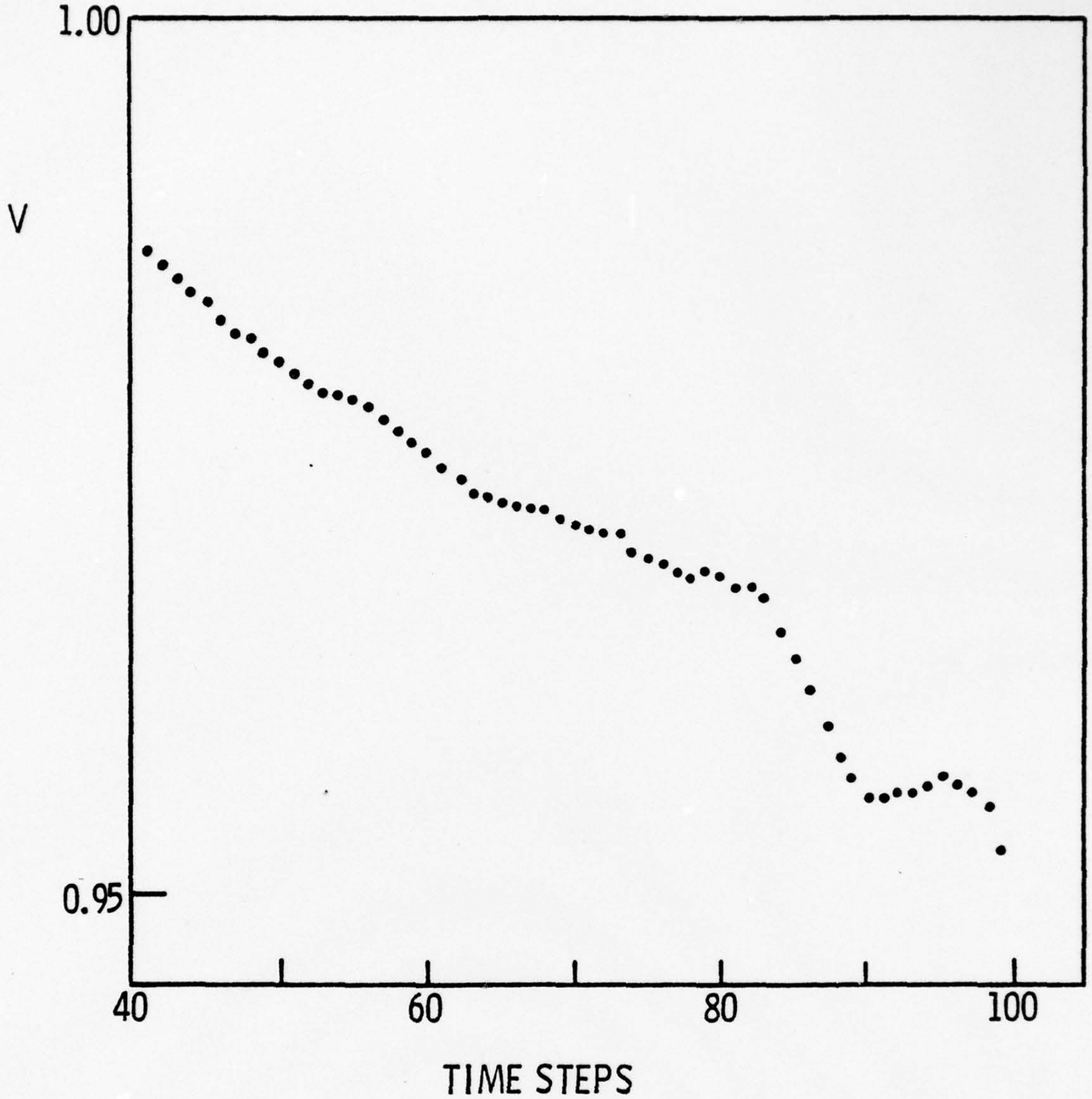


Fig. 8 Vertical Wake Velocity Decay, Reynolds Number = 250, Modified Program.

APPENDIX: PROGRAM LISTING

```
//RAMCJ126  JOB (3166,ORA316600,CO,MML,3,25,7,,,,60),"VORTEX PAIR",
//   CLASS=G,
//              MSGLEVEL=(1,1),COND=(7,LT)
//   EXEC FORTXCLG,
//   XREF=NOXREF,
//   GOTIME=5,
//   GOCORE=150K
//FORT.SYSIN  DD  *
C
C      PROGRAM TO TEST ADDITION OF VISCOSITY TO INVISCID  VORTEX PAIR
       COMMON/BOX/DT,TIME,DX,DY,H,PSI(33,63),VORT(33,63,2),U(33,63),V(33,
      163),UMAX,VMAX,CAPU,DELU(4),KONT
       COMMON/INDEX/IX,IY
       COMMON/BOX1/ FCFX(33,63),FCFY(33,63)
     C  ,VC1T(33,63),VC2T(33,63)
       COMMON /CHECK/ ICNT,IKNT,ITER
CC  **  CHANGE INPUT TO NAMELIST
       NAMELIST /INDATA/ IPRINT,IFIRST,ITAPE,TSTOP,CAPU,PSCHT,PSITST,
     C   EPS,RA,ICNT,DELCU,REDLIN,KONTF,LASTS,FACT,FACT1,FACT2,
     C   FITPX,FITPY,RUNNO
CC  **
       DIMENSION STORE(10408)
C      DIMENSION US(32,62,2),VS(32,62,2)
       DIMENSION US(1,1,1),VS(1,1,1)
       EQUIVALENCE(DT,STORE(1))
CC  **
       REAL MVJO
CC  **
       DIMENSION FVB(63),FUB(63),FVBX(63),FUBX(63),Z(63)
       DIMENSION ARG2(2),NARG(2)
C
C      PSI IS THE STREAMFUNCTION
C      U=D(PSI)/DY = X VELOCITIES
C      V = D(PSI)/DX = Y VELOCITIES
C      VORT=VORTICITY
C      ORIGIN (0.,0.) IS AT I=2,J=2
C
CC  **  FUNCTIONAL STATEMENT FOR EXTRAPOLATION OF GREEN"S FN.
CC  **
       EXTRAP(X1,X2,Y1,Y2,Y3) = X1+(X2-X1)/(Y2-Y1)*(Y3-Y1)
CC  **
CC  **  DEFINE A FUNCTIONAL STATEMENT FOR THREE POINT CENTRAL DIFFERENCE
CC  **  APPROXIMATION OF THE FIRST DERIVATIVE
CC  **
       TPCD1(XPH,XMH,H)=(XPH-XMH)/(2.*H)
CC  **
       KONT=0
       DELCU=0.
       IGRNKT=0
       IGRNH=0
       IKNT=0
       IPRTER=6
       INPUT=5
       IFILE1=11
       IFILE1=10
       IFILE2=10
       IONE=1
       ITER=0
       IX=32
       IY=62
C      IX = NUMBER OF GRID POINTS OF BOX IN X DIRECTION+2
C      IY = NUMBER OF GRID POINTS OF BOX IN Y DIRECTION+2
       IYM1=IY-1
       IXM1=IX-1
       IXP1=IX+1
```

```
      IYP1=IY+1
      IYM2=IY-2
      MVJO=32.
      CAPUST=0.0
CC  **  READ INPUT
      READ(5,5003)
 5003 FORMAT(1X"                                                          ")
      WRITE(6,5003)
      READ(5,INDATA)
CC  **  WRITE NAMELIST
      WRITE(6,INDATA)
C     PSITST=RELAXATION CONVERGENCE TEST FOR PSI ITERATION
CC  **  PSITST SET FOR INITIAL ADJUSTMENT ONLY. PROGRAM THEN          ITST
      PI=3.1415926
      RKA=3.831706
      RJOKA=-.4027594
      PIH=PI/2.
      IPRT=0
      H=1.0
C     RA = RADIUS OF CIRCLE OF ZERO PSI-LINE
      RK=RKA/RA
      RHA=RA
      PSC=RKA*RKA/(RHA*RHA)
      PIIN=1./PI
      PIITM2=-2.*PIIN
      WRITE(IPRTER,1002)        IFIRST,ITAPE,RA,TSTOP,CAPU,PSCHT,PSIT:
     1EPS
 1002 FORMAT(1H1,"THE FOLLOWING IDENTIFIES PARAMETERS FOR THIS RUN -
     1//," IFIRST = ",I2," WHERE=0 MEANS 1ST RUN",//," ITAPE = ",I2,/.
     1 RA =",E16.8,//," TSTOP = ",E16.8,//," CAPU = ",E16.8,//," PSCHT
     2  ",E16.8,//," PSITST = ",E16.8,//," EPS = ",E16.8)
      WRITE(6,6005) ICNT
 6005 FORMAT("   MAX. NO. OF TAPE RECORDS WRITTEN THIS RUN  ",I5/)
      A2=RA*RA
      U2=2.*CAPU
      G1=U2/RJOKA
C  ITAPE = 0 - PROGRAM STARTS AT T=0 AND DOES NOT WRITE FINAL ARRAYS
C  ITAPE = 1 - PROGRAM STARTS AT T=0 AND DOES     WRITE FINAL ARRAYS
C             FOR USE AS INPUT IN SUBSEQUENT RUNS
C  ITAPE = 2 - PROGRAM USES PREVIOUSLY WRITTEN FINAL ARRAYS AS STARTING P
C               AND WRITES FINAL ARRAYS FOR USE AS INPUT IN SUBSEQUENT RUNS
C  ITAPE = 3 - PROGRAM USES PREVIOUSLY WRITTEN FINAL ARRAYS AS STARTING POINT
C               AND DOES NOT WRITE FINAL ARRAYS FOR USE AS INPUT IN FUTURE RUNS
      ITYPE=ITAPE+1
C     IPRINT = VALUE OF PRINT FREQUENCY
C     IFIRST =0 FOR FIRST RUN
C            .NE.0 FOR SUBSEQUENT RUNS
C     ITAPE = TAPE OPTION ( SAME AS RADIATIVE BUOYANT CONVECTION)
C     TSTOP= TIME TO STOP RUN
C     CAPU=INPUT
CC  **  IKNT COUNTS THE NUMBER OF CALLS TO PRTOUT
CC  **  ICNT = MAX. NO. OF TAPE RECORDS TO BE WRITTEN BEFORE STOPPING
CC  **  KONT = COUNTS NO. OF TIME STEPS
CC  **  DELCU A CALCULATED CORRECTION TO CAPU
CC  **  DELU - AN ARRAY FOR CORRECTING CAPU
CC  **  IGRNKT  GREEN"S FN COUNTER
CC  **  IGRNH   GREEN"S FN INDICATOR WHICH CONTAINS THE VALUE OF IGRNKT
CC  **    THAT WAS LAST USED WHEN THE FN WAS CALCULATED
CC  **  US AND VS ARE ARRAYS FOR STORING BOUNDARY U"S AND V"S FOR EXTRAPOLATION
      IF(IFIRST.NE.0)GO TO 10
      DX=1./60.
      DY=1./60.
      GO TO 11
   10 CONTINUE
CC  **  NOTE CHANGE TO IFILE1
      IFILE1=10
```

BEST AVAILABLE COPY

```
      IYP1=IY+1
      IYM2=IY-2
      MVJO=32.
      CAPUST=0.0
CC  **  READ INPUT
      READ(5,5003)
 5003 FORMAT(1X"                                                    ")
      WRITE(6,5003)
      READ(5,INDATA)
CC  **  WRITE NAMELIST
      WRITE(6,INDATA)
C     PSITST=RELAXATION CONVERGENCE TEST FOR PSI ITERATION
CC  **  PSITST SET FOR INITIAL ADJUSTMENT ONLY. PROGRAM THEN RESETS PSITST
      PI=3.1415926
      RKA=3.831706
      RJOKA=-.4027594
      PIH=PI/2.
      IPRT=0
      H=1.0
C     RA = RADIUS OF CIRCLE OF ZERO PSI-LINE
      RK=RKA/RA
      RHA=RA
      PSC=RKA*RKA/(RHA*RHA)
      PIIN=1./PI
      PIITM2=-2.*PIIN
      WRITE(IPRTER,1002)       IFIRST,ITAPE,RA,TSTOP,CAPU,PSCHT,PSITST,
     1EPS
 1002 FORMAT(1H1,"THE FOLLOWING IDENTIFIES PARAMETERS FOR THIS RUN - ",
     1//," IFIRST = ",I2," WHERE=0 MEANS 1ST RUN",//," ITAPE = ",I2,//,"
     1 RA =",E16.8,//," TSTOP = ",E16.8,//," CAPU = ",E16.8,//," PSCHT =
     2 ",E16.8,//," PSITST = ",E16.8,//," EPS = ",E16.8)
      WRITE(6,6005) ICNT
 6005 FORMAT("   MAX. NO. OF TAPE RECORDS WRITTEN THIS RUN  ",I5/)
      A2=RA*RA
      U2=2.*CAPU
      G1=U2/RJOKA
C  ITAPE = 0 - PROGRAM STARTS AT T=0 AND DOES NOT WRITE FINAL ARRAYS
C  ITAPE = 1 - PROGRAM STARTS AT T=0 AND DOES     WRITE FINAL ARRAYS
C                FOR USE AS INPUT IN SUBSEQUENT RUNS
C  ITAPE = 2 - PROGRAM USES PREVIOUSLY WRITTEN FINAL ARRAYS AS STARTING POINT
C                AND WRITES FINAL ARRAYS FOR USE AS INPUT IN SUBSEQUENT RUNS
C  ITAPE = 3 - PROGRAM USES PREVIOUSLY WRITTEN FINAL ARRAYS AS STARTING POINT
C                AND DOES NOT WRITE FINAL ARRAYS FOR USE AS INPUT IN FUTURE RUNS
      ITYPE=ITAPE+1
C     IPRINT = VALUE OF PRINT FREQUENCY
C     IFIRST =0 FOR FIRST RUN
C           .NE.0 FOR SUBSEQUENT RUNS
C     ITAPE = TAPE OPTION ( SAME AS RADIATIVE BUOYANT CONVECTION)
C     TSTOP= TIME TO STOP RUN
C     CAPU=INPUT
CC  **  IKNT COUNTS THE NUMBER OF CALLS TO PRTOUT
CC  **  ICNT = MAX. NO. OF TAPE RECORDS TO BE WRITTEN BEFORE STOPPING
CC  **  KONT = COUNTS NO. OF TIME STEPS
CC  **  DELCU A CALCULATED CORRECTION TO CAPU
CC  **  DELU - AN ARRAY FOR CORRECTING CAPU
CC  **  IGRNKT  GREEN"S FN COUNTER
CC  **  IGRNH  GREEN"S FN INDICATOR WHICH CONTAINS THE VALUE OF IGRNKT
CC  **     THAT WAS LAST USED WHEN THE FN WAS CALCULATED
CC  **  US AND VS ARE ARRAYS FOR STORING BOUNDARY U"S AND V"S FOR EXTRAPOLATION
      IF(IFIRST.NE.0)GO TO 10
      DX=1./60.
      DY=1./60.
      GO TO 11
   10 CONTINUE
CC  **  NOTE CHANGE TO IFILE1
      IFILE1=10
```

```
CC   **
CC   **   CHANGE TO GET THE LAST "STORE" OF THOSE "STORES" PREVIOUSLY WRITTEN
CC   **   1 - 20
CC   **   2 - 40
CC   **   3 - 60
CC   **   4 - 80
CC   **   5 - 100
CC   **   6 - 120
CC   **   7 - 140
CC   **   8 - 160
CC   **   9 - 180
CC   **   10 - 200
         DO 15 I=1,LASTS
         READ(IFILE1) STORE
      15 CONTINUE
CC   **
      11 DXI=1./DX
         DYI=1./DY
         DY2=2.*DY
         DX2=2.*DX
         DXIH=.5*DXI
         DYIH=.5*DYI
         DX2I=DXI*DXI
         DY2I=DYI*DYI
         DR2=DX*DX+DY*DY
         DR2I=1./DR2
         TD=DX2I+DY2I
         TD2=2.*TD
         FMU=(DX2I/TD)*COS(PI*DX)+(DY2I/TD)*COS(PI*DY)
         A = FMU/(1.+SQRT(1.-FMU*FMU))
         PL=1.-(1.+A*A)
         PC=(1.0+A*A)*.5*DR2I
         P1=DY*DY*PC
         P2=DX*DX*PC
         P3=DY*DY*P2
CC   **
         IF(IFIRST.NE.0) GO TO 99
         DO 20 I=1,IXP1
         DO 20 J=1,IYP1
         U(I,J)=0.0
         V(I,J)=0.0
         PSI(I,J)=0.0
         VORT(I,J,1)=0.0
         VORT(I,J,2)=0.0
      20 CONTINUE
C
C        GENERATE INITIAL PSI FIELD FROM P.535 OF BATCHELOR
C
         C1=(2.0*CAPU)/(RK*RJOKA)
         TIME=0.
         DT=0.
C
C        SET UP UPPER QUADRANT OF INITIAL VALUES OF CLOSED CIRCLE AND
C        USE SYMMETRY FOR BOTTOM QUADRANT
         I1=2
         I2=(IX-2)*.5+2.0009
         J1=(IY-2)*.5+2.0009
         J2=(IY-2)*.75+2.0009
         J3=(IY-2)*.25+2.0009
         DO 30 I=I1,I2
         DO 30 J=J1,J2
         X=FLOAT(I-2)*DX
         Y=FLOAT(J-2)*DY-.5
         R=SQRT(X*X+Y*Y)
         PXR=R*R
         SINTH=X/R
```

BEST AVAILABLE COPY

```
      COSTH=-Y/R
      IF(R-RA)23,23,25
   23 ARG=RK*R
      CALL BESJ(ARG,1,BJ,.00001,IER)
      IF(IER.NE.0)WRITE(6,4999)IER,I,J,ARG,BJ
 4999 FORMAT(1H ,"IER = ",I2," I= ",I2," J= ",I2," ARG= ",E16.8,
     1 " BJ = ",E16.8)
      PSI(I,J)=-C1*BJ*SINTH
      VR=-G1*(BJ/ARG)*COSTH
      CALL BESJ(ARG,0,BJ0,.00001,IER)
      IF(IER.NE.0)WRITE(6,4989)IER,I,J,ARG,BJ0
 4989 FORMAT(1H ,"IER = ",I2," I= ",I2," J= ",I2," ARG = ",E16.8,
     1 " BJ0 = ",E16.8)
      VTHET=G1*SINTH*(BJ0-BJ/ARG)
      U(I,J)=+VR*SINTH+VTHET*COSTH
      V(I,J)=-VR*COSTH+VTHET*SINTH
      VORT(I,J,1)=PSC*PSI(I,J)
      VORT(I,J,2)=VORT(I,J,1)
      GO TO 26
   25 PSI(I,J)=-CAPU*(R-A2/R)*SINTH
      VR=-CAPU*(1.-A2/RXR)*COSTH
      VTHET=CAPU*(1.+A2/RXR)*SINTH
      U(I,J)=+VR*SINTH+VTHET*COSTH
      V(I,J)=-VR*COSTH+VTHET*SINTH
   26 K=J1-(J-J1)
      PSI(I,K)=PSI(I,J)
      U(I,K)=-U(I,J)
      V(I,K)=V(I,J)
      VORT(I,K,1)=VORT(I,J,1)
      VORT(I,K,2)=VORT(I,K,1)
   30 CONTINUE
C     INSURE THAT GRID POINTS ON CLOSED CIRCULAR BOUNDARY ARE ZERO
      WRITE(6,4979)PSI(I1,J1),PSI(I2,J1),PSI(I1,J2),PSI(I1,J3)
 4979 FORMAT(1H ,4E16.8)
      PSI(I1,J1)=0.0
      PSI(I2,J1)=0.0
      PSI(I1,J2)=0.0
      PSI(I1,J3)=0.0
C     SET UP REST OF INITIAL PSI FIELD
      I3=I2+1
      DO 40 I=I3,IXP1
      DO 40 J=J1,IYP1
      X = FLOAT(I-2)*DX
      Y=FLOAT(J-2)*DY-.5
      R = SQRT(X*X+Y*Y)
      SINTH =X/R
      COSTH=-Y/R
      RXR=R*R
      PSI(I,J)=-CAPU*(R-A2/R)*SINTH
      VR=-CAPU*(1.-A2/RXR)*COSTH
      VTHET=CAPU*(1.+A2/RXR)*SINTH
      U(I,J)=+VR*SINTH+VTHET*COSTH
      V(I,J)=-VR*COSTH+VTHET*SINTH
      K=J1-(J-J1)
      PSI(I,K)=PSI(I,J)
      V(I,K)=V(I,J)
      U(I,K)=-U(I,J)
   40 CONTINUE
      J4=J2+1
      DO 50 I=I1,I2
      DO 50 J=J4,IYP1
      X=FLOAT(I-2)*DX
      Y=FLOAT(J-2)*DY-.5
      R=SQRT(X*X+Y*Y)
      RXR=R*R
      SINTH=X/R
```

```
      COSTH=-Y/R
      PSI(I,J)=-CAPU*(R-A2/R)*SINTH
      VR=-CAPU*(1.-A2/RXR)*COSTH
      VTHET=CAPU*(1.+A2/RXR)*SINTH
      U(I,J)=+VR*SINTH+VTHET*COSTH
      V(I,J)=-VR*COSTH+VTHET*SINTH
      K=J1-(J-J1)
      U(I,K)=-U(I,J)
      V(I,K)=V(I,J)
      PSI(I,K)=PSI(I,J)
   50 CONTINUE
CC  **  CONSTANTS DEFINED ON THE BOUNDARY BG  (BEFORE GREEN)
C     PSITP=PSI(3,60)
C     PSIRT=PSI(30,32)
CC  **
      DO 60 J=1,IYP1
   60 PSI(1,J)=-PSI(3,J)
      DO 70 J=1,IYP1
      VORT(1,J,1)=-VORT(3,J,1)
      VORT(1,J,2)=VORT(1,J,1)
   70 CONTINUE
C     CALCULATE U,V FROM PSI FIELD
C     PSI IS THE STREAMFUNCTION
C     UMAX,VMAX ARE USED TO ADJUST DT
      UMAX=1.2
      VMAX=2.5
      UMAX=24.
      VMAX=50.
      UMAX=1200.
      VMAX=2500.
CC  **  RESET PSITST(PSI TEST) FOR USE IN FIRST STEP ONLY)
      IF( KONT.EQ.0) PSITST=PSITST*.1
CC  **
CC  **
CC  **
CC  **      HERE IF STARTING FROM TAPE
   99 CONTINUE
      WRITE(6,INDATA)
CC  **  GET INITIAL DETAILED PRINTOUT
C     CALL PRTOUT
CC  **
CC  **
  100 CONTINUE
CC
C
CC  **
  230 DTIX=UMAX*DYI+VMAX*DXI+TD2
      DTCRIT=1./DTIX
      DT=.8*DTCRIT
CC  **  MODIFY DT FOR FIRST STEP ONLY
      IF(KONT.EQ.0) DT=DT*.01
C     IF(KONT.EQ.0) DT=DT*.1
CC  **
  240 TIME=TIME+DT
      WRITE(6,2325)
 2325 FORMAT(///)
      WRITE(6,2300)UMAX,VMAX
 2300 FORMAT(1H ,"UMAX = ",E16.8," VMAX = ",E16.8)
      WRITE(6,2301)DT,TIME
 2301 FORMAT(1H ,"DT = ",E16.8," TIME = ",E16.8)
CC  **
CC  **  CARDS SPECIFING BOUNDARY VORTICITY VALUES GO HERE
CC  **
CC  **  CALCULTAE NEW VORTICITIES EVERYWHERE EXCEPT AT EXTERIOR POINTS
      DO 190 J=2,IY
      DO 190 I=3,IX
```

```
      V1=U(I+1,J)+U(I,J)
      IF(V1.GT.0.0)GO TO 180
      V2=V1*VORT(I+1,J,1)
      GO TO 181
  180 V2=V1*VORT(I,J,1)
  181 V3=U(I,J)+U(I-1,J)
      IF(V3.GT.0.0)GO TO 182
      V4=V3*VORT(I,J,1)
      GO TO 183
  182 V4=V3*VORT(I-1,J,1)
  183 V5=V(I,J+1)+V(I,J)
      IF(V5.GT.0.0)GO TO 184
      V6=V5*VORT(I,J+1,1)
      GO TO 185
  184 V6=V5*VORT(I,J,1)
  185 V7=V(I,J)+V(I,J-1)
      IF(V7.GT.0.0)GO TO 186
      V8=V7*VORT(I,J,1)
      GO TO 187
  186 V8=V7*VORT(I,J-1,1)
  187 VC1=DXIH*(V2-V4)
      VC2=DYIH*(V6-V8)
      VC1T(I,J)=VC1
      VC2T(I,J)=VC2
      VC=2.*VORT(I,J,1)
      VC4=DX2I*(VORT(I+1,J,1)-VC+VORT(I-1,J,1))
      VC5=DY2I*(VORT(I,J+1,1)-VC+VORT(I,J-1,1))
CC  **
CC  **
CC  **  SIGNU - INDICATES WHAT VALUE TO USE IN CORRECTING FOR FALSE VISCOSITY
CC  **  SIGNV - INDICATES WHAT VALUE TO USE IN CORRECTING FOR FALSE VISCOSITY
      SIGNU=SIGN(.5,-U(I,J))
      SIGNV=SIGN(.5,-V(I,J))
      IF(U(I,J).EQ.0) SIGNU=0.
      IF(V(I,J).EQ.0) SIGNV=0.
CC  **
      FVCX=   SIGNU*DX*(U(I,J)*VC4+TPCD1(U(I+1,J),U(I-1,J),DX) *
     C                        TPCD1(VORT(I+1,J,1),VORT(I-1,J,1),DX))
      FVCY=SIGNV*DY*(V(I,J)*VC5+TPCD1(V(I,J+1),V(I,J-1),DY)*
     C       TPCD1(VORT(I,J+1,1),VORT(I,J-1,1),DY))
CC  **
CC  **  FITPX   AND FITPY    ARE FITTING PARAMETERS RELATED TO THE
CC  **       FALSE VISCOSITY CORRECTION TERMS IN X AND IN Y  AND ARE
CC  **  NORMALLY SET EQUAL TO 1
CC  **
CC  **  $ $ $ $
      FCFX(I,J)=FVCX
      FCFY(I,J)=FVCY
      IF(ABS(FVCX).GT.FITPX*ABS(VC1)) FCFX(I,J)=FCFX(I,J)*1.E20
      IF(ABS(FVCY).GT.FITPY*ABS(VC2)) FCFY(I,J)=FCFY(I,J)*1.E20
      IF(ABS(FVCX).GT.FITPX*ABS(VC1)) FVCX=SIGN(VC1,FVCX)
      IF(ABS(FVCY).GT.FITPY*ABS(VC2)) FVCY=SIGN(VC2,FVCY)
C     VORT(I,J,2)=VORT(I,J,1 )+DT*((VC4+VC5-VC1-VC2)
..8 **  NOTE  V:S.OU] ,\RM ]E,8,04:\:?  8
      VORT(I,J,2)=VORT(I,J,1 )+DT*((0.0+0.0-VC1-VC2)
     C       +FVCX  +FVCY)
  190 CONTINUE
      DO 107 I=3,IX
CC  **  CALCULATE VORTICITY ALONG TOP EXTERIOR LINE
      VORT(I,IYP1,2)=3.*(VORT(I,IYP1-1,2)-VORT(I,IYP1-2,2))
     C    +VORT(I,IYP1-3,2)
CC  **  CALCULATE VORTICITY ALONG BOTTOM EXTERIOR LINE
      VORT(I,1,2)=3.*(VORT(I,2,2)-VORT(I,3,2))+VORT(I,4,2)
  107 CONTINUE
CC  **  CALCULATE VORTICITY ALONG RIGHT EXTERIOR LINE
      DO 108 J=1,IYP1
```

```
      VORT(IXP1,J,2)=3.*(VORT(IXP1-1,J,2)-VORT(IXP1-2,J,2))   +
     C    VORT(IXP1-3,J,2)
  108 CONTINUE
CC  **
      DO 191 J=1,IYP1
      VORT(1,J,2)=-VORT(3,J,2)
  191 CONTINUE
CC  **  BY-PASS GREEN"S FN. EXTRAPOLATION
CC  **        WHEN STARTING FROM TIME ZERO
CC  **
CC  **
      IF(KONT.LE.20) GO TO 195
CC  **
CC  **
CC  **  TO AVOID SKIPPING
      IF(KONT.GE.0) GO TO 195
CC  **
CC  **  IGRNKT IS COUNT CONTROL FOR CALCULATION
CC  **  OF GREEN"S FUNCTION
CC  **
CC  **  ALWAYS PERFORM CALCULATION WHEN STARTING OFF
CC  **
      IGRNKT=IGRNKT+1
CC  **
      IF(MOD(IGRNKT,3).EQ.0.AND.DT.LT.1.5E-4) GO TO 575
  195 CONTINUE
      WRITE(6,6010)
 6010 FORMAT("     GREENS FUNCTION USED DURING THIS STEP    ")
C     IMPOSE GREENS FUNCTIONS TO OBTAIN U,V ON COUNDARIES
C
C     SEARCH FOR NON=ZERO VORTICITIES
C
      NYU=0
      NYL=0
      NXR=0
      NXL=3
      DO 90 J=3,IYM1
      IF(ABS(VORT(3,J,1)).LE.EPS)GO TO 90
      NYL=J
      GO TO 91
   90 CONTINUE
   91 DO 92 J=1,IYM1
      K=IY-J
      IF(ABS(VORT(3,K,1)).LE.EPS)GO TO 92
      NYU=K
      GO TO 93
   92 CONTINUE
   93 DO 96 J=NYL,NYU
      DO 94 I=3,IXM1
      IF(ABS(VORT(I,J,1)).GT.EPS)GO TO 94
      NX=I-1
      GO TO 95
   94 CONTINUE
   95 IF(NX.GT.NXR)NXR=NX
   96 CONTINUE
      WRITE(IPRTER,1050)NXL,NXR,NYL,NYU
 1050 FORMAT(1H0," NXL = ",I2," NXR = ",I2," NYL = ",I2," NYU = ",I2)
C
C     LIMITS OF NON=ZERO VORTICITY AREA HAVE BEEN ESTABLISHED
C
C     CALCULATE U,V ON Y=0 AND Y=1 BOUNDARY LINES
C
      ARG2(1)=0.0
      ARG2(2)=IY-2
      NARG(1)=2
      NARG(2)=IY
```

```
      DO 510 L=1,2
      YB=ARG2(L)
      LL=NARG(L)
C     DO 500 I1=3,IXM1
CC ** CHANGE I1 BOUNDS TO INCLUDE CORNER POINTS
CC **
CC ** JSKIP - 1 OR 2 DEPENDING WHETHER GREEN"S FN. IS INTERPOLATED
CC **      AT EVERY OTHER POINT OR CALCULATED AT EVERY POINT
CC **
      JSKIP=2
CC **
      DO 500 I1=2,IX,JSKIP
      K=1
      XB=FLOAT(I1-2)
      XB2=XB*XB
      FVBX(1)=0.0
      FUBX(1)=0.0
      DO 420 J=NYL,NYU
      K=K+1
      YP=FLOAT(J-2)
      FVB(1)=0.0
      FUB(1)=0.0
      FVB(NXR)=0.0
      FUB(NXR)=0.0
      CY=YB-YP
      CY2=CY*CY
      DO 410 I=3,NXR
      XP=FLOAT(I-2)
      DENOM=((XB-XP)*(XB-XP)+CY2  )*((XB+XP)*(XB+XP)+CY2  )
      FVB(I-1)=VORT(I,J,1)*DXI*XP*((XB2  -XP*XP-CY2  )/DENOM)
      FUB(I-1)=(VORT(I,J,1)*DXI*XB*XP*CY)/DENOM
  410 CONTINUE
      CALL QSF(DX,FVB,Z,NXR)
      FVBX(K)=Z(NXR)
      CALL QSF(DX,FUB,Z,NXR)
      FUBX(K)=Z(NXR)
  420 CONTINUE
      K=K+1
      FVBX(K)=0.0
      FUBX(K)=0.0
      CALL QSF(DY,FVBX,Z,K)
      V(I1,LL)=Z(K)*PIIN+CAPU
      CALL QSF(DY,FUBX,Z,K)
      U(I1,LL)=Z(K)*PIITM2
  500 CONTINUE
CC ** FIRST SET OF U AND V MODIFIERS GO HERE
CC ** FIRST SET

      DO 505 I1=3,IXM1,2
      U(I1,LL)=(U(I1-1,LL)+U(I1+1,LL))*.5
  505 V(I1,LL)=(V(I1-1,LL)+V(I1+1,LL))*.5
CC **
  510 CONTINUE
C
C     CALCULATE U,V ON X=.5 LINE
C
      ARG2(1)=0.0
      ARG2(2)=IX-2
      NARG(1)=2
      NARG(2)=IX
      LL=2
      I1=NARG(LL)
      XB=ARG2(LL)
      DO 550 L=3,IYM1,JSKIP
      K=1
      FVBX(1)=0.0
```

```
      FUBX(1)=0.0
      YB=FLOAT(L-2)
      DO 540 J=NYL,NYU
      K=K+1
      YP=FLOAT(J-2)
      CY=YB-YP
      CY2=CY*CY
      DO 530 I=3,NXR
      XP=FLOAT(I-2)
      DENOM=((XB-XP)*(XB-XP)+CY2  )*((XB+XP)*(XB+XP)+CY2  )
      FVB(I-1)=VORT(I,J,1)*DXI*XP*((XB2  -XP*XP-CY2  )/DENOM)
      FUB(I-1)=(VORT(I,J,1)*DXI*XB*XP*CY)/DENOM
  530 CONTINUE
      CALL QSF(DX,FVB,Z,NXR)
      FVBX(K)=Z(NXR)
      CALL QSF(DX,FUB,Z,NXR)
      FUBX(K)=Z(NXR)
  540 CONTINUE
      K=K+1
      FVBX(K)=0.0
      FUBX(K)=0.0
      CALL QSF(DY,FVBX,Z,K)
      V(II,L)=Z(K)*PIIN+CAPU
      CALL QSF(DY,FUBX,Z,K)
      U(II,L)=Z(K)*PIITM2
  550 CONTINUE
CC  **
CC  **  GREENS FUNCTION TEST
      IYM2=IYM1-1
CC  **  SECOND SET OF
CC  **  U AND V MODIFIERS GO HERE
CC  **  SECOND SET
      DO 555 L=4,IYM2,2
      U(IX,L)=(U(IX,L-1)+U(IX,L+1))*.5
      V(IX,L)=(V(IX,L-1)+V(IX,L+1))*.5
  555 CONTINUE
CC  **
      GO TO  (588,588),IONE
CC
C
  575 CONTINUE
CC  **
CC  **
      GO TO (590,590),IONE
CC  **
CC  **
CC  **  CHANGE V(AFTER) TO V(BEFORE)
CC  **
      NDIF=IY-2
      DO 580 I=2,IX
      DO 580 J=2,62,NDIF
      V(I,J)=V(I,J)-DELCU
  580 CONTINUE
      DO 585 J=3,IYM1
      V(IX,J)=V(IX,J)-DELCU
  585 CONTINUE
CC  **  END CHANGE
  588 CONTINUE
  590 CONTINUE
CC  **
C     CALCULATE PSI ON BOUNDARIES AT TOP,BOTTOM AND AT RIGHT
C
      PSI(3,2)=-DX*V(2,2)
      PSI(3,IY)=-DX*V(2,IY)
      DO 700 I=3,IXM1
      PSI(I+1,IY)=PSI(I-1,IY)-DX2*V(I,IY)
```

```
      PSI(I+1,2)=PSI(I-1,2)-DX2*V(I,2)
  700 CONTINUE
      PSI(IX,IYM1)=2.*PSI(IX,IY)-PSI(IXM1,IY)+DX*V(IX,IY)-DY*U(IX,IY)
CC  **  CHANGE TO REFLECT VORTICITY ON THE BOUNDARY
     C      -.5*DX*DX*VORT(IX,IY,2)
      DO 710 J=3,IYM1
      K=IYP1-J
      PSI(IX,K)=PSI(IX,K+2)-DY2*U(IX,K+1)
  710 CONTINUE
  250 ITER=0
  260 DPSIMX=0.
      PSIMAX=0.
      ITER=ITER+1
CC  **  TEST ITER TO PREVENT RUN AWAY ITERATIONS
      IF(ITER.GE.35) CALL PRTOUT
      DO 270 J=3,IYM1
      DO 270 I=3,IXM1
      PSI(I,J)  =RL*PSI(I,J)+P1*(PSI(I+1,J)+PSI(I-1,J))+P2*(PSI(I,J+1)+
     1PSI(I,J-1))+VORT(I,J,2)*P3
  270 CONTINUE
      DO 280 I=3,IXM1
      DO 280 J=3,IYM1
      PS=PSI(I,J)
      PSI(I,J)  =RL*PSI(I,J)+P1*(PSI(I+1,J)+PSI(I-1,J))+P2*(PSI(I,J+1)+
     1PSI(I,J-1))+VORT(I,J,2)*P3
      DPSI=ABS(PS-PSI(I,J))
CC  **
      IF(DPSI.LE.DPSIMX)GO TO 288
      DPSIMX=DPSI
CC  **  FIND COORDINATES OF DPSIMX
      IIH1=I
      IIH2=J
CC  **  END FIND
  288 CONTINUE
      IF(ABS(PSI(I,J)).LE.PSIMAX)GO TO 280
      PSIMAX=ABS(PSI(I,J))
  280 CONTINUE
CC  **  CHANGE J BOUND TO GET CORNER POINT
      DO 290 J=1,IY
      PSI(1,J)=-PSI(3,J)
  290 CONTINUE
CC  **  PSITST TEST
CC  **
      WRITE(6,6001) IIH1,IIH2
 6001 FORMAT(40X,2I10)
      WRITE(6,6002) DPSIMX,PSIMAX
 6002 FORMAT(2F20.4)
CC  **
      IF(DPSIMX.GT.PSITST*PSIMAX)GO TO 260
C
CC  **  RESET PSI TEST VALUE FOR ALL SUBSEQUENT TIME STEPS
      IF(KONT.EQ.0) PSITST=PSITST*10.
CC  **
CC  **  WRITE INDATA REFLECTING CHANGE OF PSI TEST
      IF(KONT.EQ.0) WRITE(6,INDATA)
CC  **
      DO 281 I=1,IXP1
      DO 281 J=1,IYP1
      VORT(I,J,1)=VORT(I,J,2)
  281 CONTINUE
C
CC  **  COUNT NO. OF TIME STEPS
      KONT=KONT+1
      WRITE(6,6022) KONT
 6022 FORMAT("    NUMBER OF THIS TIME STEP =  ",I5)
C     DETERMINE CHANGE IN CAPU = CHANGE IN Y OF STAGNATION POINT
```

```
CC   **   (AT MAX PSI) DIVIDED BY DT
CC   **
C
         MVI=0
         MVJ=0
         PSIMX=0.
         DO 600  I=3,IX
         DO 600  J=3,IY
         IF(PSI(I,J).LE.PSIMX) GO TO 600
         MVI=I
         MVJ=J
         PSIMX=PSI(I,J)
  600    CONTINUE
CC   **   FIND MAX PSI IN THE NEIGHBORHOOD OF PSI(MVI,MVJ)
CC   **
CC   **   USE THE BEST NINE POINTS AND THE SECOND BEST NINE POINTS
CC   **
CC   **
         MD=SIGN(1.,(PSI(MVI+1,MVJ) -PSI(MVI-1,MVJ)))
         KD=0
  610    CONTINUE
         A01=PSI(MVI,MVJ+1)-PSI(MVI,MVJ)
         A0M1=PSI(MVI,MVJ-1)-PSI(MVI,MVJ)
         A10=PSI(MVI+1,MVJ)-PSI(MVI,MVJ)
         AM10=PSI(MVI-1,MVJ)-PSI(MVI,MVJ)
CC
         B5=.25*(PSI(MVI+1,MVJ+1)
        1        -PSI(MVI-1,MVJ+1)
        1        -PSI(MVI+1,MVJ-1)
        1        +PSI(MVI-1,MVJ-1))
         B1=.5*(A10-AM10)
         B2=.5*(A01-A0M1)
         B3=.5*(A10+AM10)
         B4=.5*(A01+A0M1)
         DELTA=4.*B3*B4-B5*B5
         DELX=-(2.*B1*B4-B2*B5)/DELTA
         DELY=(-B2-B5*DELX)*.5/B4
         DELPSI      =B1*DELX+B2*DELY+B3*DELX**2+B4*DELY**2+B5*DELX*DELY
         PSIMX=PSI(MVI,MVJ)+DELPSI
         WRITE(6,6025) PSI(MVI,MVJ),PSI(MVI,MVJ+1),PSI(MVI,MVJ-1),
        C      PSI(MVI+1,MVJ),PSI(MVI-1,MVJ)
         WRITE(6,6025) PSI(MVI+1,MVJ+1),PSI(MVI+1,MVJ-1),PSI(MVI-1,MVJ-1),
        C  PSI(MVI-1,MVJ+1)
         WRITE(6,6025) DELX,DELY,DELPSI
 6025    FORMAT(5E20.7)
         IF(KD.EQ.1) GO TO 650
         KD=1
         W1=1.-ABS(DELX)
         DELX0=MVI+DELX
         DELY0=DELY
         PSIMX0=PSIMX
         MVI=MVI+MD
         IF(KD.EQ.1) GO TO 610
  650    CONTINUE
         DELX=MVI+DELX
CC   **
CC   **
CC   **
CC   **   THESE EXPRESSIONS WEIGHT THE RESULT TOWARD THE MOST CENTRAL ONE
CC   **
         DELXP=W1*DELX0+(1.-W1)*DELX
         DELX=DELXP-MVI
         PSIMX=PSIMX+MD*DELX*(PSIMX-PSIMX0)
CC   **   THIS EXPRESSION WEIGHTS THE RESULT TOWARD THE MOST CENTRAL ONE
         DELYPP=MVJ+DELY+MD*DELX *(DELY-DELY0)
CC   **
```

```
CC   **
C        WRITE(6,6025) A01,A0M1,A10,AM10
         WRITE(6,6025) A01,A0M1,A10,AM10
C        WRITE(6,6025) B1,B2,B3,B4,B5
         WRITE(6,6025) B1,B2,B3,B4,B5
         WRITE(6,3010) PSIMX,DELXP,DELYPP
 3010 FORMAT("0","    MAX PSI =",E16.8,"  AT I=  ",F8.4,"  J=  ",F8.4)
CC   **
CC   **
CC   **  2 POINT PREDICTOR
CC   **  4 POINT CORRECTOR
         IF(KONT.GT.3) GO TO 620
         DELU(KONT)=DELYPP-REDLIN
         GO TO 670
  620 CONTINUE
         DELU(1)=DELU(2)
         DELU(2)=DELU(3)
         DELU(3)=DELYPP-REDLIN
         WRITE(6,6030) (DELU(I),I=1,3)
 6030 FORMAT(//"    YS",5X,4F15.7//)
         IF(KONT.LT.20) GO TO 670
         YPRED=2*DELU(3)-DELU(2)
         YDES=(32.*DELU(3)-12.*DELU(2))/27.
CC   **  FACT IS A PARAMETER TO CONTROL OSCILATIONS IN THE CONTROL SYS.
CC   **  AS IS FACT1
CC   **  AS IS FACT2
CC   ** THE FOLLOWING FACTS ARE FOR CAPU(0)=1000.
C        FACT=.1
C        FACT1=.303
C        FACT2=1.
CC   **
CC   **  THE FOLLOWING FACTS ARE FOR CAPU(0)=20.
C        FACT=.2
C        FACT1=0
C        FACT2=4
CC   **
         DELCUF=.5*(YDES-YPRED)*DY/DT/FACT2- FACT*(DELU(3)-DELU(2))*DY/DT
        C      -FACT1*(DELU(3)-2.*DELU(2)+DELU(1))*DY/DT
CC   **
CC   **
C        DELCU=DELCUC+DELCUF
CC   **  NOTE ONLY DELCUF USED
         DELCU=DELCUF
         CAPU=CAPU+DELCU
  670 CONTINUE
         WRITE(6,3013) CAPU
 3013 FORMAT(//"    CAPU=",E16.7)
         WRITE(6,3011) DELCU
 3011 FORMAT(1H ,"     DELTA CAPU =",E16.7)
         CAPUST=DELCU
CC   **
         WRITE(IPRTER,1008)ITER
 1008 FORMAT(1H ," ITER = ",I3    )
CC   **
CC   **
CC
CC   ALTER PSI TO REFLECT CHANGE IN FREE STREAM VELOCITY (CAPUST)
CC
         DO 680 I=3,IX
         DO 680 J=2,IY
         PSI(I,J)=PSI(I,J)-CAPUST*DX*(I-2)
  680 CONTINUE
         DO 682 J=2,IY
         PSI(1,J)=-PSI(3,J)
  682 CONTINUE
CC   **
```

```
CC  **  ALTER V ON BOUNDARY TO REFLECT CHANGE IN FREE STREAM VELOCITY
        DO 690 I=2,IX
        V(I,2)=V(I,2)+CAPUST
        V(I,IY)=V(I,IY)+CAPUST
    690 CONTINUE
        DO 692 J=3,IYM1
        V(IX,J)=V(IX,J)+CAPUST
    692 CONTINUE
CC  **  END ALTER
CC  **
CC  **  U AND V GENERATED BY FCD
CC  **
        UMAX=0.0
        VMAX=0.0
        DO 110 I=2,IXM1
        DO 110 J=3,IYM1
        U(I,J)=DYIH*(PSI(I,J+1)-PSI(I,J-1))
        UTRY=ABS(U(I,J))
        IF(UTRY.GT.UMAX)UMAX=UTRY
        V(I,J)=-DXIH*(PSI(I+1,J)-PSI(I-1,J))
        VTRY=ABS(V(I,J))
        IF(VTRY.GT.VMAX)VMAX=VTRY
    110 CONTINUE
CC  **
CC  **  EXTRAPOLATE U AND V TO EXTERIOR LINE
CC  **              .......ALONG TOP
        DO 112 I=3,IX
        U(I,IYP1)=3.*(U(I,IYP1-1)-U(I,IYP1-2))+U(I,IYP1-3)
        V(I,IYP1)=3.*(V(I,IYP1-1)-V(I,IYP1-2))+V(I,IYP1-3)
CC  **      ....AND ALONG BOTTOM
        U(I,1)=3.*(U(I,2)-U(I,3))+U(I,4)
        V(I,1)=3.*(V(I,2)-V(I,3))+V(I,4)
    112 CONTINUE
CC  **  EXTRAPOLATE U AND V TO EXTERIOR LINE
CC  **  ..........ALONG RIGHT SIDE
        DO 114 J=1,IYP1
        U(IXP1,J)=3.*(U(IXP1-1,J)-U(IXP1-2,J))+U(IXP1-3,J)
        V(IXP1,J)=3.*(V(IXP1-1,J)-V(IXP1-2,J))+V(IXP1-3,J)
    114 CONTINUE
CC  **
        IF(KONT.LE.20) GO TO 850
CC  **  TO AVOID SKIPPING
        IF(KONT.GE.0) GO TO 850
CC  **
CC  **  USE EXTRAPOLATION FOR U AND V ON THE BOUNDARY
CC  **  EXTRAPOLATION ASSUMES THE FOLLOWING SEQUENCE OF EVENTS
CC  **  1 SAVE U(B),V(B)   (AFTER CHANGE OF REFERENCE PT.)
CC  **  2 SAVE U(B),V(B)   (AFTER CHANGE OF REFERENCE PT.)
CC  **  4 EXTRAPOLATE DELTA CAPU
CC  **  3 EXTRAPOLATE U(B),V(B)     IF DELTA TIME IS SMALL ENOUGH
CC  **  5 IN PLACE OF GREEN"S FN. ALTER V(B) BY EXTRAPOLATED DELTA CAPU
CC  **
        IF(MOD(IGRNKT,3).EQ.0) GO TO 850
CC  **
        K=MOD(IGRNKT,3)
        NDIF=IY-2
        DO 760 I=2,IX
```

```
      DO 760 J=2,IY,NDIF
      US(I,J,K)=U(I,J)
      VS(I,J,K)=V(I,J)
  760 CONTINUE
CC  **  SAVE TIME
      IF(K.EQ.1) TS=TIME
CC  **  SAVE DELCU
      IF(K.EQ.1) DELCUS=DELCU
CC  **
      DO 765 J=2,IY
      US(IX,J,K)=U (IX,J)
      VS(IX,J,K)=V (IX,J)
  765 CONTINUE
      WRITE( 6,6026) K
 6026 FORMAT("   U(B),V(B) SAVED",I5)
CC  **
CC  **
      IF(MOD(IGRNKT,3).NE.2) GO TO 850
CC  **  CALCULATE DELTA TIME FOR EXTRAPOLATION USE
      DTIX=UMAX*DYI+VMAX*DXI+DT2
      DTCRIT=1./DTIX
      DT=.8*DTCRIT
      TIMET=TIME+DT
CC  **  END CALCULATE
      NDIF=IY-2
      DO 780 I=2,IX
      DO 780 J=2,IY,NDIF
C     WRITE(6,6019) I,US(I,J,1),US(I,J,2)
      U(I,J)=EXTRAP(US(I,J,1),US(I,J,2),TS,TIME    ,TIMET)
      V(I,J)=EXTRAP(VS(I,J,1),VS(I,J,2),TS,TIME    ,TIMET)
C     WRITE(6,6019) I,U(I,J),V(I,J)
 6019 FORMAT(I5,4F10.2)
  780 CONTINUE
      DO 785 I=3,IYM1
      U(IX,I)=EXTRAP(US(IX,I,1),US(IX,I,2),TS,TIME,TIMET)
      V(IX,I)=EXTRAP(VS(IX,I,1),VS(IX,I,2),TS,TIME,TIMET)
C     WRITE(6,6019) I,U(32,I),V(32,I)
  785 CONTINUE
      WRITE(6,6020)
 6020 FORMAT("   U(B),V(B) EXTRAPOLATED")
      DELCU=EXTRAP(DELCUS,DELCU,TS,TIME    ,TIMET)
  850 CONTINUE
CC
  210 IPRT=IPRT+1
C     IF((IPRT/IPRINT)*IPRINT.NE.IPRT)GO TO 220
CC  **  KONTF = FINAL COUNT
CC  **
CC  **  STORE ON TAPE HERE
C     IF(MOD(KONT,20).EQ.0) WRITE(10) STORE
CC  **
CC  **  CALL PRINT OUT HERE
 4    :A::4PR::UT4
CC  **
      IF(KONT.EQ.KONTF) STOP 33
  220 CONTINUE
CC  **
      IF(KONT.GE.0) GO TO 100
C     IF(TIME.LT.TSTOP)GO TO 100
CC  **
      IF((IPRT/IPRINT)*IPRINT.EQ.IPRT)GO TO 225
C     WRITE OUT FINAL ARRAYS
      CALL PRTOUT
      GO TO (222,221,222,221),ITYPE
  221 CONTINUE
```

```
      WRITE(IFILE2)STORE
 2210 WRITE(IPRTER,1004)
 1004 FORMAT(1H ," FINAL ARRAYS ON DISK - TERMINATION NORMAL")
      STOP
  222 CONTINUE
 2220 WRITE(IPRTER,1009)
 1009 FORMAT(1H ," FINAL ARRAYS NOT ON DISK - TERMINATION NORMAL ")
      STOP
  225 CONTINUE
      GO TO (2220,2210,2220,2210),ITYPE
  300 WRITE(IPRTER,1005)
 1005 FORMAT(1H ,"TOP LEFT VALUE OF PSI ON BOUNDARY HAS CHANGED TOO MUCH
     1  EXECUTION TERMINATED")
      CALL PRTOUT
      STOP
  310 WRITE(IPRTER,1006)
 1006 FORMAT(1H ,"CENTER RIGHT VALUE OF PSI ON BOUNDARY HAS CHANGED TOO
     1 MUCH  EXECUTION TERMINATED")
      CALL PRTOUT
      STOP
 3000 CONTINUE
      WRITE(IPRTER,1007)IER,ARG
 1007 FORMAT(1H ," ERROR RETURN FROM SUBROUTINE BESJ-- IER = ",I3,
     1    "ARG = ",E16.8)
      S T O P S
      END
      SUBROUTINE PRTOUT
      COMMON/BOX/DT,TIME,DX,DY,H,PSI(33,63),VORT(33,63,2),U(33,63),V(33,
     163),UMAX,VMAX,CAPU,DELU(4),KONT
      COMMON/BOX1/ FCFX(33,63),FCFY(33,63)
     C  ,VC1T(33,63),VC2T(33,63)
      COMMON/INDEX/IX,IY
      COMMON /CHECK/ ICNT,IKNT,ITER
      IKNT=IKNT+1
      IXP1=IX+1
      IYP1=IY+1
C     PRINT PSI,U,V,VORT
      WRITE(6,1000)
 1000 FORMAT(1H1,"PRINT OF PSI,U,V,VORT FOLLOWS - ")
      WRITE(6,999)TIME
  999 FORMAT(1H ,"TIME = ",E16.8)
      ISKIP=1
      M=1
      DO 100 J=M,IYP1,ISKIP
      L=IYP1-J+1
      WRITE(6,1001)L
 1001 FORMAT(1H0,"ROW = ",I3)
      WRITE(6,1002)(PSI(I,L),I=M,IXP1,ISKIP)
      WRITE(6,1002)(  U(I,L),I=M,IXP1,ISKIP)
      WRITE(6,1002)(  V(I,L),I=M,IXP1,ISKIP)
      WRITE(6,1002)(VORT(I,L,2),I=M,IXP1,ISKIP)
      WRITE(6,1002) (FCFX(I,L),I=M,IXP1,ISKIP)
      WRITE(6,1002) (FCFY(I,L),I=M,IXP1,ISKIP)
      WRITE(6,1002) (VC1T(I,L),I=M,IXP1,ISKIP)
      WRITE(6,1002) (VC2T(I,L),I=M,IXP1,ISKIP)
 1002 FORMAT(1H ,10E13.5)
  100 CONTINUE
      IF(ITER.GE.35) STOP 2
      IF(IKNT.GE.ICNT) STOP 1
      RETURN
      END
```

BEST AVAILABLE COPY

```
//LKED.SYSLIB DD DSN=SYS1.TSOFLIB,DISP=SHR
//              DD DSN=FORT.SSPLIB,DISP=SHR
//GO.SYSIN  DD  *
 DEL X=1/60 T(1)=T(1)*.01 CAPU(0)=1000
 $INDATA
 IPRINT=20,
 ITAPE=1,
 TSTOP=.1,
 PSCHT=.5,
 PSITST=.001,EPS=.001,RA=.25,
 IENT=2,
 ICNT=5,
 FITPX=1.,FITPY=1.,
 FITPX=1.E50,FITPY=1.E50,
 REDLIN=32.0006,
 CAPU=1000.,
 RUNNO=11.9,FACT=.1,FACT1=.303,FACT2=2.34,
 KONTF=4,
 LASTS=0,
 IFIRST=1,
 IFIRST=0,
 $END
```